VFC DLOM
CALCULATOR

EMPIRICAL RESEARCH REGARDING DISCOUNTS FOR LACK OF MARKETABILITY

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Introduction by Francis A. Longstaff, Ph.D.

The question of how to value illiquid investments that cannot be traded continuously is one of the most challenging issues facing academic researchers and industry practitioners. The reason for this is that the lack of marketability takes us well outside standard paradigms in financial economics such as the notion of efficient markets, portfolio choice, and the usual risk and return tradeoffs that underlie much of modern investment theory.

Marc Vianello's book “Empirical Research Regarding Discounts for Lack of Marketability" is an impressive effort to bring a rigorous and comprehensive data-based perspective to addressing these issues. The book begins with a thorough review of the historical research on the topic and provides valuable insights about the scope and reliability of the evidence. The book then moves on to an insightful analysis of the strengths and weaknesses of existing models of the discount for lack of marketability. What makes this analysis particularly valuable is the depth of knowledge and practical experience the author brings to the task. Finally, the book offers a number of carefully considered extensions to existing models, demonstrates how these can be implemented in practice, and evaluates their performance using objective empirical standards.

This book makes great strides in helping us understand the nature of the discount for lack of marketability phenomenon and offers us valuable perspectives on how to address the associated challenges of valuation.

Francis A. Longstaff, Ph.D.
Allstate Chair in Insurance and Finance
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Introduction by Michael Gregory, ASA, CVA, MBA

With a dedication to improve analytics for business valuers, Marc Vianello has diligently and passionately conducted his research on Discount for Lack of Marketability over many years. As a result of his research he has shed light on the shortcomings of many existing models, and he has developed a tool that is based on real world data and that has been accepted by the courts.

After a careful look at the literature followed by a critique of major sources commonly used by business valuers, the author presents very significant findings. An analysis of the data with graphs, charts and statistical measures presents reasons to question currently accepted approaches. Starting with Longstaff model probability is incorporated into a new model considering the mean and standard deviation of market timing and volatility. The author presents how to obtain these measures from existing data sources (systemic) and provides the business valuer with insights with how to consider the application of non-systemic professional judgment.

From the text the author states, "Double probability DLOMs calculated using the Longstaff formula provided values most consistent with the empirical evidence provided by the discounts of corresponding restricted stock transactions. The calculated DLOMs should be considered systematic. The currently available empirical information supports the conclusion that double probability DLOMs calculated using the VFC Longstaff methodology results in reliable estimates of systematic DLOM." This is very significant in that no other source can make such a claim.

This is a tool that no business valuer should be without.

Michael Gregory, ASA, CVA, MBA
Former IRS Engineering Territory Manager
Champion of the *IRS DLOM Job Aid*

About the Author

Marc Vianello, CPA, is the owner of Vianello Forensic Consulting, LLC. He is accredited in the business valuation (ABV) and certified in financial forensics (CFF by the American Institute of Certified Public Accountants.

Mr. Vianello graduated in 1975 from the University of Missouri, *cum laude*, with a Bachelor's Degree in Business Administration and a major in Accountancy. A former financial statement auditor, tax consultant, public utility rate consultant, and entrepreneur, Mr. Vianello has spent most of his professional career providing expert testimony in highly complex commercial litigation.

Previously Published Content

Some of the content in this book reflect Mr. Vianello's previously published articles listed below. The concepts underlying the articles have been corroborated by the research presented in this book. Previously presented thoughts regarding adjustments to the Longstaff formula are superseded by this book.

"New Insight into Calculating Discounts for Lack of Marketability," Financial Valuation and Litigation Expert, Issue 11, February/March 2008; republished by CPA Expert, May 2008.

"Restructuring the Levels of Value," BVR's Guide to Discounts for Lack of Marketability, 2009 Edition.

"Calculating DLOM Using the VFC Longstaff Methodology," BVR's Guide to Discounts for Lack of Marketability, 2009 Edition.

"The Specific Company Risk of Abnormal Levels of Debt," Valuation Strategies, September/October 2010 Edition.

"The Marketing Period of Private Sales Transactions," Business Valuation Update, Vol. 16, No. 12, December 2010

"The Marketing Period of Private Sale Transactions: Updated for Sales through 2010," Business Valuation Update, Vol. 17, No. 11, November 2011.

"Rebutting Critics of the Longstaff DLOM Methodology," Business Valuation Update, Vol. 18, No. 9, September 2012.

"Why Do Private Firms Linger on the Selling Block?" Business Valuation Update, Vol. 19, No. 10, October 2013.

"How Probability Affects Discounts for Lack of Marketability," Business Valuation Update, Vol. 20, No. 7, July 2014.

"Using Restricted Stock and Pre-IPO Studies for Quantifying DLOM - Two Ways of Saying I Don't Know?" Valuation Strategies, September/October 2014 Edition.

"Calculating Probability Based DLOMs," Valuation Strategies, November/December 2014 Edition.

"Probability Based Estimation and the DLOM Calculation," QuickRead, August 19, 2015 (a National Association of Certified Valuators and Analysts publication).

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PREFACE

The appropriate amount of discount for lack of marketability ("DLOM") has long been critical for valuation professionals, investors in and issuers of illiquid securities, financial statement issuers and auditors, the courts, and others. The determination of an appropriate discount has been extensively discussed and debated. Yet, to this author's knowledge, no one has heretofore made the intensive empirical study necessary to actually justify a DLOM methodology using stringently-defined, objective data. That is the purpose of this research.

The research presented herein has been years in development. The analyzed data provides extensive insight into the market evidence of liquidity discounts. And the data supports and challenges different methodologies for determining DLOMs. The study results strongly favor basing DLOM estimation on probability-based option modeling as opposed to other commonly available means.

The data used in this research is necessarily limited to that available at the time the analysis was done. It uses extensive transactional data possessed, or otherwise accessible, by the author. Much of it should be updated as new data becomes available. In particular, transactions that have been added to the Pluris®, Stout (formerly FMV Opinions®), *BIZCOMPS®*, and *DealStats®* (formerly *Pratt's Stats*) databases. Those additions are a matter for a future supplement of this research. The author invites those issuers to participate in his research efforts.

Two other limitations affected our research. First, our analyses were hampered by a lack of restricted stock issuer daily price data more than 10 or 20 years old depending on the price data source. Daily price data is necessary to determine price probability volatilities. 1,687 restricted stock transactions escaped analysis because daily price history before the transaction dates was not available to the author. More price data may be available from other sources not currently available to the author. The Center for Research in Security Prices ("CRSP") is one potential source. Second, much of the restricted stock transactional discount data available through Pluris® is tainted with warrants. 1,867 transactions escaped analysis because of the manner in which Pluris® values warrants and, therefore, restricted stock discounts. Repricing the warrants using the Black-Scholes formula might yield analytically viable data, which the author invites Pluris® to do, and to provide.

The author invites qualified interested parties to participate in his continuing DLOM research.

Finally, the author extends his gratitude to the Business Valuation Committee of the American Institute of Certified Public Accountants for its assistance, recommendations, and encouragement in completing this book.

Marc Vianello, CPA, ABV, CFF

July 1, 2019

Chapter 1
LIQUIDITY AND LEVELS OF VALUE

Liquidity represents the ability to sell an investment quickly when the investor decides to sell. Conversely, *lack* of liquidity, although having many causes,[1] has the cost of failing to realize gains or failing to avoid losses on an investment during the period in which the investor is offering it for sale. With that understanding, discounts for lack of marketability ("DLOM") should reflect the illiquidity cost of the investment—its value volatility—during the period of time that it is being marketed for sale.

The valuation profession has written volumes about "levels of value" over the years. One concept has placed a higher value on "control" than on "liquidity." The relative levels of value under this "Control Dominant" structure are presented as –

Control Value

Difference reflects the value of control

Publicly Traded Value

Difference reflects the value of marketability

Non-Marketable Minority Value

Under the Control Dominant concept, the "control premium" regularly measured by MergerStat® has been offered as proof that Control Value is worth more than Publicly Traded Value, assuming that all other things are equal. But does the Control Dominant concept hold if the interpretation given to MergerStat's® "control premium" is incorrect, and that it instead measures the discount (or a portion of the discount) imposed by non-strategic investors on poorly run public companies? Another example of potentially faulty Control Dominant logic is the notion that Publicly Traded Value exclusively represents the return expectations of minority stakeholders. But does the Control Dominant view hold if instead the returns realized on publicly-traded securities represent risk adjusted rates at which the expectations of all marginal non-strategic investors are equalized based on the expected cash flows of the enterprise? Others hold the view that Control Value equates to Publicly Traded Value, giving "control" a presumption of virtually immediate liquidity. But does this alternative hold considering the time periods necessary to sell a controlling interest and associated transaction costs?

When comparing the relative values of controlling and minority interests in the same privately-held company, it is easy to intuit that the ability to control the enterprise is worth more

[1] A non-exhaustive list of causes of illiquidity includes lack of buyers, excessive pricing, transaction costs, business complexity, income stream risk, and much more.

than not having that ability. Hence, all other things equal, Control Value is logically greater than Minority Value. But that logic does not lead to a conclusion that Control Value is greater than Publicly Traded Value on a per share basis. Imagine a controlling interest in a publicly traded company. The controlling investor owning a comparatively large or unregistered block of stock is exposed to the same price volatility as the minority investors, but is denied the opportunity to as quickly dispose of his interest in the company. This realization suggests that liquidity (because it offers the ability to protect the value of one's investment) is worth more than control share-for-share.

Let us explore the factors that result in different levels of value. When comparing the value drivers of well run publicly traded businesses (value based on non-controlling stock trades) and well run privately controlled businesses (value based on the entirety), we find that the only real difference is liquidity or its lack:

Public Companies	**Private Companies**
Earnings / Cash Flow	Earnings / Cash Flow
Growth potential	Growth potential
Industry Risk	Industry Risk
Size Risk	Size Risk
Market Fluctuations	Market Fluctuations
Liquidity	*No Liquidity*

With the understanding that liquidity represents the ability to sell an investment quickly without price impact and little transaction cost when the investor decides to sell in order to lock in gains or to avoid losses, then, assuming everything else to be equal, the *inability* to quickly liquidate a controlling interest in a publicly traded company suggests that it is worth less per share than the liquid minority shares. That observation leads initially to this Restructured View of the levels of business value:

Publicly Traded Value

Difference reflects the economic risk of lack of marketability

Illiquid Control Value

Difference reflects the economic risk of lack of control

Non-Marketable Minority Value

The basis of this Restructured View is straightforward. First, the investment returns of publicly traded companies should be viewed as "public company returns" not as "marketable minority returns." For well run companies that are operating optimally for their shareholders, there should be no economic difference (aside from compliance costs) between public company operating results and those accruing to controlling interests of otherwise identical private companies – the material perquisites of control have been squeezed out of the public companies. Poorly run companies (i.e. those not operating optimally for their shareholders) have difficulty maintaining shareholder value and raising new capital.[2] Consequently, publicly traded companies that are not optimized have difficulty attracting capital in the form of fractional ownership.

Second, Strategic Value does not enter into the determination of required rates of return, which are based on the prices of shares actually traded. Although an increase in stock price may be offered to existing shareholders as an inducement to sell, the actual benefits of a strategic acquisition accrue to the merged company as revenues are enhanced and expenses are minimized. Such effects are reflected in the income statement and cash flow of the enterprise as a whole and contribute to increased value that is shared by all post-acquisition ownership interests. Such effects are not suggestive of the notion that Strategic Value is worth more than Publicly Traded Value. Although a value may be derived from a strategic opportunity, it does not mean that the opportunity is worth more than the value of liquidity once the opportunity is realized. After all, once the opportunity is realized, the merged-company owners are subject to return volatility just as the owners of publicly traded securities are. This price risk applies to all owners of the enterprise, whether they hold registered or unregistered shares, restricted or unrestricted shares, and controlling or minority shares.

There are well run publicly traded companies and well run privately held companies. There are also poorly run companies of both types. When a public company is acquired at a premium above its publicly traded value it is a reflection of the perception that the acquired company is not maximizing its economic opportunities and shareholder value. Well-run publicly traded companies (i.e. those that are maximizing their economic opportunities and shareholder value) are not taken private—they are too expensive. This is not to say that an acquirer cannot simply overpay or that two well-run public companies cannot merge to take advantage of market opportunities that have nothing to do with management deficiencies. Obviously, such acquisitions happen. But these scenarios nonetheless reflect expectations of post-acquisition benefits not being realized by the acquired company. Accordingly, the "premium" observed when publicly traded companies are taken private reflects the anticipation that some nature of inefficiencies in the acquired company can and will be eliminated. For these reasons, the so-called "control premium studies" are misused when used to suggest that control is worth more than liquidity.

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[2] Some have observed that cash flows underlying Publicly Traded Value minus the benefits of liquidity equate to those underlying Illiquid Control Value minus the benefits of control. While conceptually legitimate, there is no known empirical means of equating the benefits of liquidity and the benefits of control, and the two benefits may be far from equal. This negates the usefulness of the observation.

Consider these thoughts: (1) Risk adjusted rates of return are fungible.[3] (2) There is a transaction cost to becoming and continuing as a publicly traded company. This creates a disincentive that can only be justified by (a) greater access to capital, and (b) the "pop" in value that the pre-IPO owners receive when their business goes public. (3) If control were worth more than liquidity, then the owners of privately held businesses would have a further disincentive to going public. (4) If control were more valuable than liquidity, then there would be no public companies.[4] (5) If control were worth more than liquidity, then large private equity firms such as Blackstone and KKR would never convert to publicly traded companies. It seems counter-intuitive that control should be viewed as equal in value to—or even more valuable than—liquidity.

Under otherwise identical circumstances, any given investment should have a greater value if it is immediately marketable than if it is not. Why is this so? Because liquidity allows the investor to avoid the economic risks of illiquidity.

The notion of a control premium vis-à-vis public company values is economically illogical. Such premiums mathematically equate to lower rates of return. But since it is expected that it would take longer to sell a controlling interest in an optimally run private company than the comparable interest in an otherwise identical public company, the required rate of return of the private company investor should be greater, not lower, than that of the public company investor. Thus, private company values should reflect a discount, not a premium, relative to comparable public company values.

Figure 1.1 presents the Restructured View of value in greater dimension. The depiction shows how well run and poorly run private companies relate to each other and how the opportunity to realize strategic value (including market synergies) arises from the conversion of poorly run firms into firms that hopefully will be well run. The depiction also demonstrates that all privately held companies—even controlling interests—are subject to the cost of illiquidity.[5] Even

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[3] Eric W. Nath, ASA, and M. Mark Lee, CFA "Acquisition Premium High Jinks," 2003 International Appraisal Conference, American Society of Appraisers; Eric W. Nath, ASA, "How Public Guideline Companies Represent 'Control' Value for a Private Company," Business Valuation Review, Vol. 16, No. 4, December 1997; and Eric W. Nath, "Control Premiums and Minority Discounts in Private Companies," Business Valuation Review, Vol. 9, No. 2, June 1990.

[4] Id.

[5] It has been suggested by some practitioners that discounts for lack of liquidity should not be applied to controlling interests because the earnings and cash flow of the company offset the discount while it is being held for sale. This argument fails because (1) it relies on a flawed view of the levels of value that ignores that (a) rates of return derive from analysis of publicly traded stocks, and (b) liquidity is the only driver of value of publicly traded companies not present in otherwise identical privately held companies; (2) the economic circumstance of holding period earnings and cash flow also exists for minority interests; and (3) the holding period earnings and cash flow of both controlling interest and minority interest investments are necessarily already included in the capitalized or discounted values of the investments.

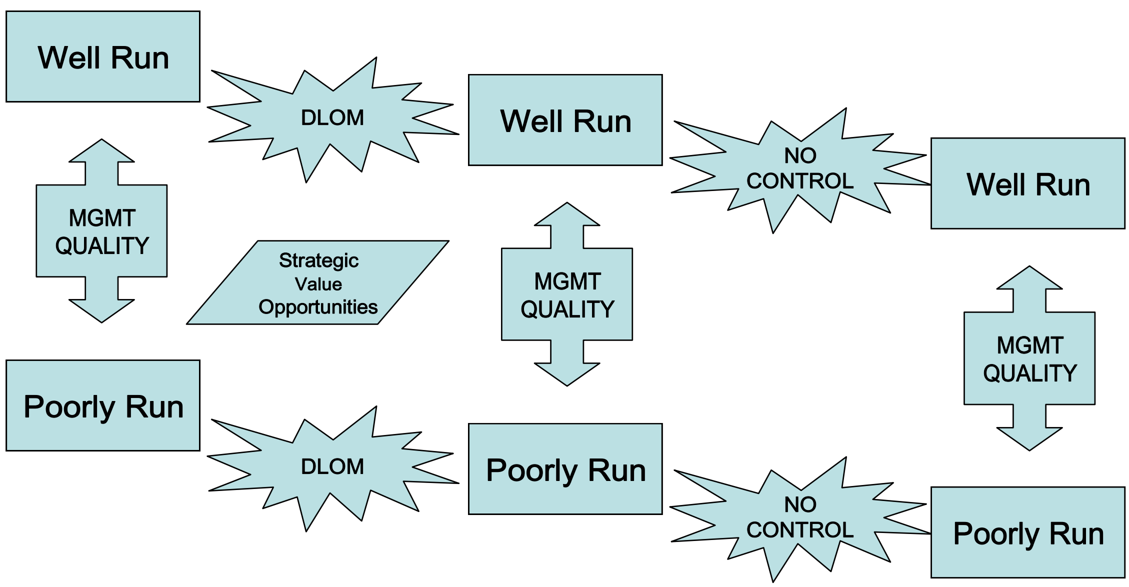
assuming all other things being equal, it simply takes longer to sell a controlling interest in a privately held business than it takes to sell an interest in a comparable publicly traded company. Minority interests in privately held companies are worth proportionately less than controlling interests for two reasons: (1) such minorities generally lack the ability of controlling owners to realize the perquisites of ownership, and (2) the economic risks of lack of control result in longer periods of time to sell minority interests than it takes to sell the controlling interest in the same private company.

Figure 1.1

ALTERNATIVE VIEW OF THE LEVELS OF BUSINESS VALUE

Public Companies | **Controlling interests in Private Companies** | **Minority Interests in Private Companies**

INCREASING MARKETING PERIODS ➔➔➔



Whether a private company can be sold via public offering is a critical valuation consideration. Chapter 6 [to be renumbered] discusses the empirical evidence of the time required to sell private company and to obtain SEC approval for a public equity offering. Table 1.1 summarizes the average marketing times by broad Standard Industrial Classification. Equating S-1 filing with a private company brokerage listing, Table 1.1 shows that it typically takes about twice the time to complete a private company sale than to obtain approval for a public offering, while taking about 4.5 times as long to sell a private company as it takes to process the average non-IPO offering. The shorter marketing periods for companies for which a public offering is a viable alternative should result in lower discounts for lack of marketability if all other things are equal. Of course, many things necessary for a public filing may be completed in advance, and many things necessary for a private sale may occur after brokerage listing. And some large companies may be able to be sold privately within a public offering time frame. Such circumstances would narrow the valuation differences between the two marketing paths. Nevertheless, there must be a value increment that incentivizes public registration or there would be no publicly traded companies.

Table 1.1
Average Number of Days to Complete a Private Sale or Public Offering

SIC Code Range	Average Private Company Completed Sale Days	Average Processing Days for Successful Public Offerings			Private Sale to Public Offering Time Factor		
		All	IPOs	Non-IPOs	All	IPOs	Non-IPOs
0000-0999	210	123	130	40	1.7	1.6	5.2
1000-1999	256	103	117	50	2.5	2.2	5.1
2000-2999	230	95	112	43	2.4	2.1	5.3
3000-3999	235	97	109	49	2.4	2.1	4.8
4000-4999	211	100	111	51	2.1	1.9	4.1
5000-5999	210	93	109	34	2.3	1.9	6.2
6000-6999	219	103	113	52	2.1	1.9	4.2
7000-7999	205	93	105	43	2.2	1.9	4.7
8000-8999	202	96	111	50	2.1	1.8	4.1
9000-9999	63	0	0	0	n/a	n/a	n/a
All industries	213	97	110	46	2.2	1.9	4.6

Number of

Table 1 (Continued) SIC Code Range	Successful Private Sales	Number of Successful Public Offerings: All	IPOs	Non-IPOs
0000-0999	545	13	12	1
1000-1999	887	271	214	57
2000-2999	1,016	788	598	190
3000-3999	962	959	769	190
4000-4999	575	421	344	77
5000-5999	7,967	436	342	94
6000-6999	353	711	595	116
7000-7999	4,643	1,256	998	258
8000-8999	1,359	302	231	71
9000-9999	2	0	0	0
All industries	18,309	5,157	4,103	1,054

Chapter 2

THE INTERRELATIONSHIP OF EMPIRICAL STUDIES OF DISCOUNTS AND LIQUIDITY

Conventional business valuation has used the well-publicized results of restricted stock studies, pre-IPO studies, and registered versus unregistered stock studies to effectively guess at appropriate DLOM percentages to use in their valuation reports. Understandably, such subjective means of applying the traditional approaches have been broadly unsatisfactory to the valuation community and the courts.

Figure 2.1

The Interrelationship of Observed Risk, Liquidity, and Discounts

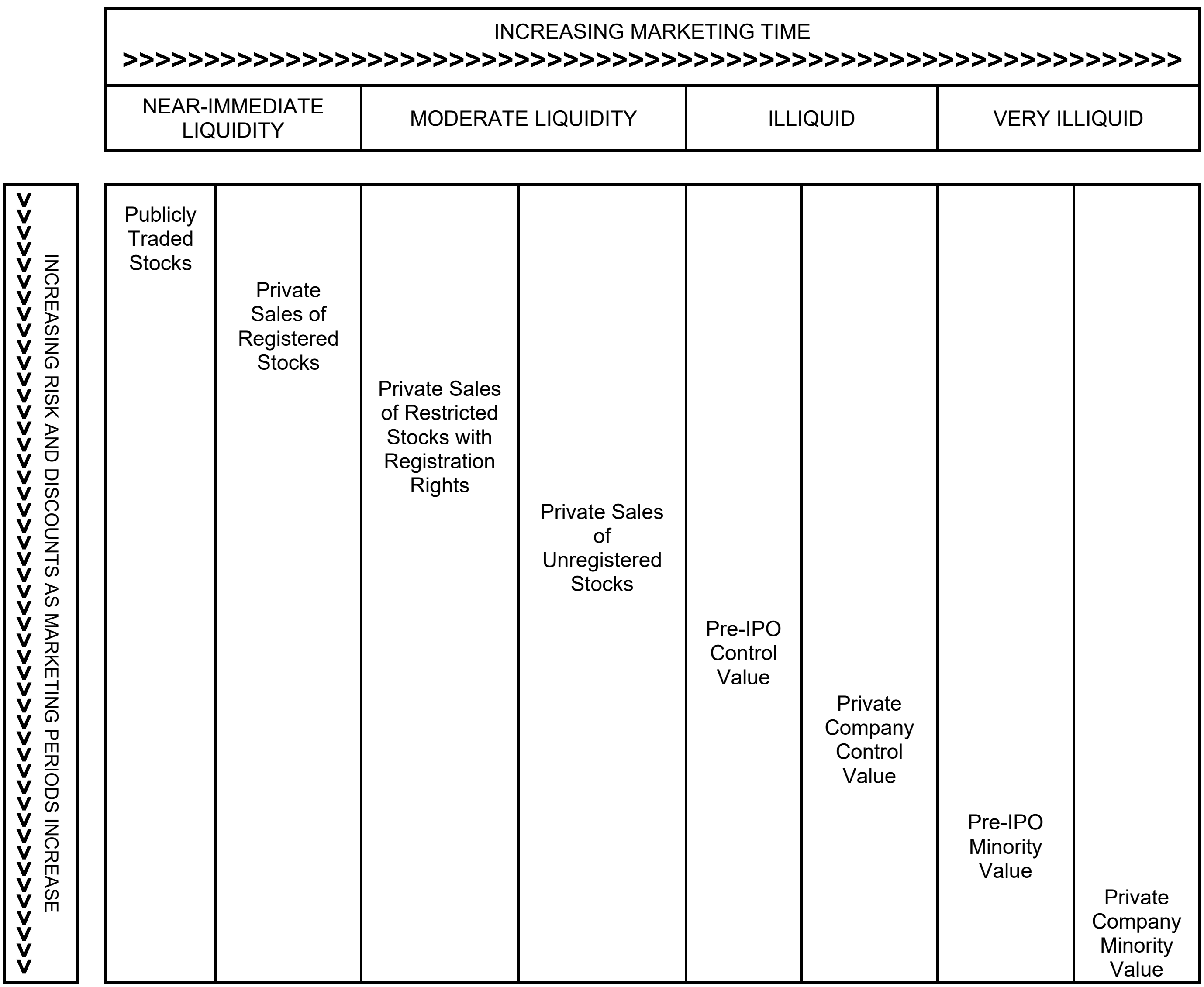


Figure 2.1 presents a stratification of the types of empirical studies that researchers have performed to explore the cost of illiquidity. The study types are shown in theoretical relative position based on marketing time and volatility assuming all other aspects of investment as equal. Although Figure 2.1 shows a stair-stepping of the studies, it is not the intent of the presentation to suggest that linear reduction of value results.[6] The presentation is, instead, intended to enhance understanding of what the various studies are measuring, how they interrelate, and the extent to which they meet the needs of business valuation discount analysis.

- Publicly traded companies are the standard against which all of the studies measure results and from which rates of return are calculated. Interests in publicly traded companies are worth more than interests in identical privately held companies because they can be sold immediately to realize gains and to avoid losses, while interests in privately held companies cannot. Although there are costs to being a publicly traded company, the assumption is that such costs are more than offset by a lower cost of capital. If this were not inherently true then there would be no economic justification for incurring those costs.
- Private sales of publicly registered stocks typically involve large blocks of stock that could be sold into the public marketplace, but which would materially adversely affect stock prices if the entire block were to be dumped into the market at once. Avoiding that price effect results in an extended period of time to liquidate the investment position in the public market during which time the investor is subject to market risk. Negotiating a private sale of the block can accelerate liquidating the position, but requires a buyer with the wherewithal to purchase the block. Such buyers can reasonably expect a price discount relative to the publicly traded price. Although private sales of large blocks of registered stocks may somewhat mitigate the market risk by potentially shortening selling periods, the risk does not go away. The buyer of the block assumes the risks, in turn, of having to sell to another qualified buyer or slowly feeding the block into the public market. These risks require compensation by means of a discount (i.e. DLOM).
- Private sales of restricted stocks in public companies have the same price risks as private sales of large blocks of registered stocks, but have the additional risk of being locked out of the public market for specific periods of time or being subject to restrictive “dribble out” rules. Accordingly, restricted stocks often can only be sold quickly in private sale transactions, which take longer than it does to sell unrestricted

[6] The relative value of specific companies should be considered in the framework of Figure 1.1, which provides an understanding of why, for example, public companies are sometimes taken private.

stocks in the public market.[7] The result is that a restricted registered stock is worth less than an unrestricted stock in the same company because of the greater market risk associated with the extended marketing period.

- Private sales of unregistered stocks in public companies typically involve large blocks of stock. They are worth less than equivalent blocks of registered stock (whether restricted or unrestricted) in the same publicly traded company because there is a cost for eventual registration that directly lowers value and can dissuade potential buyers.[8] The result is relatively greater uncertainty, relatively longer time to market the interest, and relatively greater exposure to the risks of the marketplace.
- Pre-IPO private sales of controlling interests should have relatively longer marketing periods than for private sales of unregistered stocks in public companies, because the fact and timing of the IPO event can be uncertain. Furthermore, low pre-IPO stock sales prices may reflect compensation for services rendered. This author is not aware of any studies that specifically address discounts observed in sales of controlling interests in pre-IPO companies.
- Private sales of controlling interests in a company that has no expectation of going public should be worth less than an otherwise identical company with an anticipated IPO event. The marketing period for a business with an anticipated IPO event should be shorter than the marketing period of a business that is not anticipating such an event.
- Pre-IPO sales of non-controlling interests in a company planning an IPO event should be worth less than the controlling interest in the same company even without the planned IPO. The inability to control whether the planned IPO goes forward should result in greater uncertainty and a longer marketing period to liquidate the investment than would be experienced by the controlling investor. Low pre-IPO share prices may also reflect compensation for services rendered.
- Non-controlling interests in private companies require greater discounts than all of the preceding circumstances because the relative risks of lacking control cause the

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[7] Some restricted stocks cannot be sold at all for contractually determined periods of time. Such investments have even greater economic risks than those merely subject to the “dribble out” rules.

[8] This discount is considered by Mukesh Bajaj, David J. Dennis, Stephen P. Ferris and Atulya Sarin in their paper “Firm Value and Marketability Discounts.” Their study isolates the value of liquidity by comparing the stock sales of 88 companies that had sold both registered and unregistered stock private offerings. This approach does not, however, address the discount applicable to the additional time it takes to sell controlling or minority interests in private companies. Instead, it measures the value of stock registration. *See* Section IV.C of “Firm Value and Marketability Discounts.”

period of time to liquidate the position to be potentially much longer than for the controlling interest in the same company or for otherwise comparable minority positions in firms with a planned IPO event.

Chapter 3

THE EMPIRICAL STUDIES OF RESTRICTED STOCKS AND INITIAL PUBLIC OFFERINGS ARE INADEQUATE FOR ESTIMATING DLOM

Restricted stock and pre-IPO studies have been used to quantify DLOM since the early 1970s. Despite making a good case for the need for a DLOM when valuing an investment that is not immediately marketable, the study results are unreliable for calculating the DLOM applicable to a particular valuation engagement for a variety of reasons discussed below.

Although the empirical studies of marketability discounts provide a wealth of empirical evidence of the discounts that market participants demand on risky assets, the studies have limited utility to the appraiser opining on the fair market value of a business interest. Several authors have noted, for example, that most publicly traded firms do not issue restricted stock. This dearth necessitates study samples of limited sizes, in limited industries, with data spread over long periods of time. The result has been substantial standard errors in discount estimates.

The restricted stock studies measure the difference in value between a publicly traded stock with and without a time restriction on sale. Left unanswered is whether there is a difference between the restricted stock value of a publicly traded company and the value of the same company if it were not publicly traded at all.

The pre-IPO studies reflect substantial standard errors in their estimates for similar reasons, but are also distorted by the fact that the studies necessarily are limited to successful IPOs; there are no post-IPO stock prices for failed IPOs. The discounts observed in the pre-IPO studies may also reflect uncertainty about whether the IPO event will actually occur,[9] when the IPO event will occur, at what price the event will occur, and whether the pre-IPO price reflects compensation for any reason.

It should be noted that all of the companies in the restricted stock and pre-IPO studies are, in fact, publicly traded. But essentially none of the privately held companies that are the subject of business valuations have a foreseeable expectation of ever going public. Accordingly, the circumstances of the privately held companies are highly distinguishable from those of the publicly traded companies that are the subjects of the studies. Thus, the pre-IPO studies are of dubious value for determining the DLOM of privately held companies.

Bajaj, et al., studied the difference in value observed when comparing private sales of registered stocks with private sales of unregistered stocks in the same publicly traded company. The result is a measure of the value of registration; it is not a measure of liquidity, much less a measure of DLOM for an interest in a privately held company. The DLOM applicable to the unregistered shares of a public company is not limited to the direct cost of registration and

[9] Research by Vianello Forensic Consulting, LLC indicates that only about 30% of all SEC S-1 filings are eventually approved for public offering. *See* Chapter 6 at Section 3.

applicable transaction costs. It also includes the indirect cost represented by the time it will take to obtain registration. Both costs are reasonably estimable whether the company is publicly traded or privately held, but those costs are likely much greater for the stock of a private company than for the unregistered stock of a publicly traded company.

Section 1 – Restricted Stock Studies

Restricted stocks are public company stocks subject to limited public trading pursuant to Securities and Exchange Commission ("SEC") Rule 144. Restricted stock studies attempt to quantify DLOM by comparing the sale price of publicly traded shares to the sale price of otherwise identical marketability-restricted shares of the same company.[10] The median and average ("mean") marketability discount and related standard deviation (where available) determined by some of the published restricted stock studies follows in Table 3.1:[11]

Table 3.1
PUBLISHED RESTRICTED STOCK STUDIES

	Number of Observations	Reported Median	Reported Mean	Reported Standard Deviation	Discount Range	
					Low	High
SEC overall average (1966-June 1969)	398	24%	26%	n/a	(15%)	80%
Milton Gelman (1968-1970)	89	33%	33%	n/a	<15%	>40%
Robert E. Moroney (1969-1972)	146	34%	35%	18%	(30%)	90%
J. Michael Maher (1969-1973)	34	33%	35%	18%	3%	76%
Robert R. Trout (1968-1972)	60	n/a	34%	n/a	n/a	n/a
Stryker / Pittock	28	45%	n/a	n/a	7%	91%
Willamette Management Associates (1981-1984)	33	31%	n/a	n/a	n/a	n/a
Silber (1981-1988)	69	n/a	34%	24%	(13%)	84%
Stout(Hall / Polacek) (1979-1992)	100+	n/a	23%	n/a	n/a	n/a
Stout(1991-1992)	243	20%	22%	16%	n/a	n/a
Management Planning, Inc. (1980-1995)	53	25%	27%	14%	3%	58%
Management Planning, Inc. (1980-1995)	27	9%	12%	13%	n/a	n/a
BVR (Johnson) (1991-1995)	72	n/a	20%	15%	(10%)	60%
Columbia Financial Advisors (1996-April 1997)	23	14%	21%	n/a	0.8%	68%
Columbia Financial Advisors (May 1997-1998)	15	9%	13%	n/a	0%	30%

[10] Internal Revenue Service, *Discount for Lack of Marketability Job Aid for IRS Valuation Professionals*, pages 12 and 13

[11] Page 28, "Valuation Discounts and Premiums," Chapter Seven, Fundamentals, Techniques & Theory, National Association of Certified Valuators and Analysts (NACVA), supplemented by other sources.

In 1997, the SEC reduced the two-year restriction period of Rule 144 to one year.[12] Subsequently, Columbia Financial Advisors, Inc. completed a study that analyzed restricted stock sales from May 1997 through December 1998. This study found a range of discounts from 0% to 30%, and a mean discount of 13%.[13] The conclusion reached from this study is that shorter restriction periods result in lower discounts. In 2008, the SEC further reduced the Rule 144 restriction period to six months.[14] According to the IRS, no restricted stock studies have been published that reflect the six-month holding period requirement.[15] Considering the age of the restricted stock studies, the Rule 144 transitions, and changes in market conditions, concluding that a DLOM derived from the above studies ignores current market data and conditions seems unavoidable.

Appraisers face other serious problems when relying on these studies. Because the sample sizes of the restricted stock studies are small, most involving less than 100 individual data points, the reliability of the summary statistics is subject to considerable data variation.[16] This fact alone calls the reliability of the studies into question. But the studies also report high standard deviations, as shown in the table above, indicating the probability of a very broad range of underlying data points. Relying solely on the averages of these studies is, therefore, likely to lead the appraiser to an erroneous DLOM conclusion.[17]

The graph below was prepared using the Oracle *Crystal Ball* software to model a 200,000-trial normal statistical distribution based on the reported means and standard deviations of the 146-observation Moroney study. It discloses that the potential range of discounts comprising the 35% mean discount of this study is from *negative* 44.5% to positive 113.9%--broader than the observed range, which is from negative 30% to positive 90%.

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[12] Securities and Exchange Commission, *Revisions to Rules 144 and 145*, Release No. 33-8869; File No. S7-11-07, at pages 7 and 13, et seq. http://www.sec.gov/rules/final/2007/33-8869.pdf

[13] Pratt, Shannon P., Business Valuation Discounts and Premiums, page 157, J. Wiley & Sons, Inc. (2001).

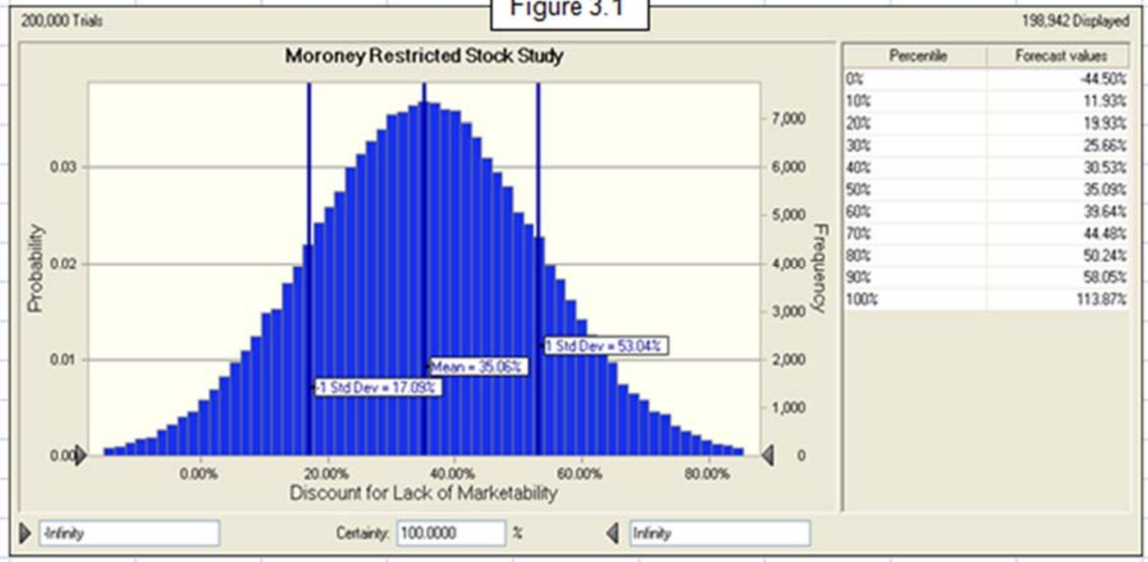
[14] Securities and Exchange Commission, *Revisions to Rules 144 and 145*, Release No. 33-8869; File No. S7-11-07, at pages 13, et seq. http://www.sec.gov/rules/final/2007/33-8869.pdf

[15] Internal Revenue Service, *Discount for Lack of Marketability Job Aid for IRS Valuation Professionals*, page 17.

[16] Id. page 15.

[17] Id. page 17.

Figure 3.1

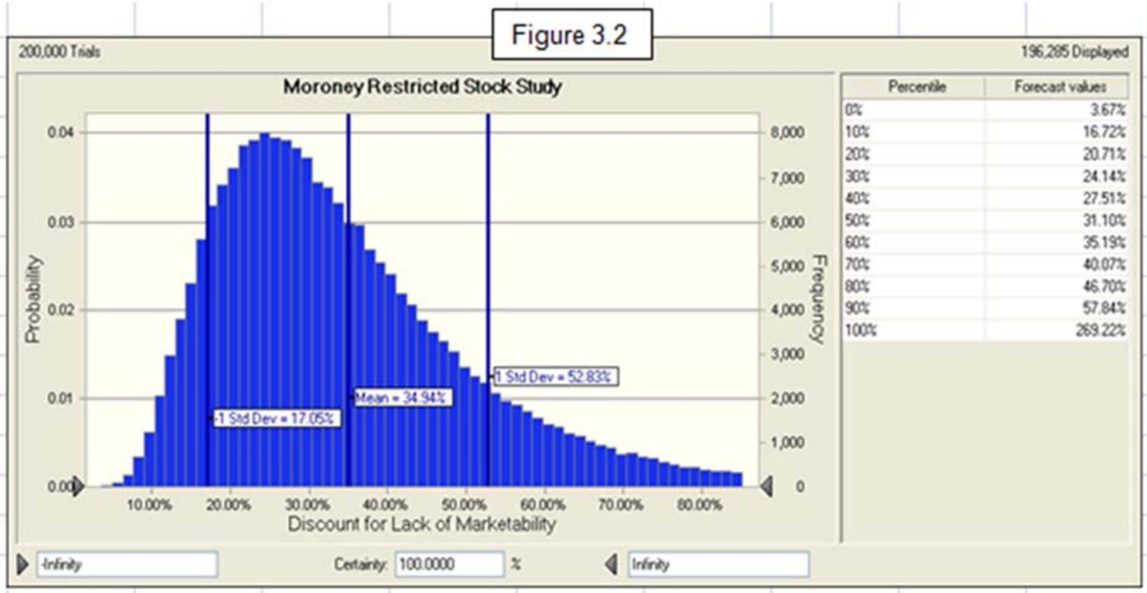


Applying the same normal distribution analysis to the Maher, Silber, and Management Planning studies, we find:

- The potential range of discounts comprising the Maher study average of 35.0% is from *negative* 41.0% to positive 110.6%.
- The potential range of discounts comprising the Silber study average of 34.0% is from *negative* 75.8% to positive 138.0%.
- The potential range of discounts comprising the 49-observation Management Planning study is from *negative* 32.5% to positive 83.1%.
- The potential range of discounts comprising the 20-observation Management Planning study is from *negative* 29.9% to positive 83.7%.

Common sense tells one that a DLOM cannot be negative. Therefore, normal statistical distribution cannot be the appropriate assumption regarding the distribution of the population of restricted stocks. A log-normal distribution must instead be assumed for the population. Using *Crystal Ball* with the log-normal assumption and 200,000 trials resulted in the graph below. It discloses that the log-normal range of discounts comprising the Moroney study is from 3.7% to 269.2% with a median discount of 31.1%. Approximately 60% of probable outcomes occur below the study mean.

Figure 3.2



Applying the same log-normal distribution analysis to the Maher, Silber, and Management Planning studies, we find:

- The log-normal range of discounts comprising the Maher study is from 4.0% to 276.6% with a median discount of 31.2%. Approximately 60% of probable outcomes occur below the study mean.
- The log-normal range of discounts comprising the Silber study is from 2.0% to 472.8% with a median discount of 27.8%. More than 60% of probable outcomes occur below the study mean.
- The log-normal range of discounts comprising the Management Planning study is from 2.7% to 233.1% with a median discount of 25.0%. Approximately 60% of probable outcomes occur below the study mean.

There may be myriad causes for such extreme results, such as issuer stock price volatility, long marketing times or periods of restriction, large blocks of stock, and regulatory hurdles, among other things that affect the perceived investment risks, but, regardless, the appraiser is left with two problems. First, what should be done about the fact that some portion of the distribution continues to imply a DLOM greater than 100%? Can that simply be ignored? Is some form of adjustment required? Second, with 60% or more of the predicted outcomes occurring below the reported means of the studies, what is the justification for assuming a DLOM based on a study's mean (or an average of studies' means)? These issues, the inability of the studies to reflect market dynamics (past or present), the inability to associate the studies with a specific valuation date, and the inability to associate the study results to a valuation subject with any specificity, seriously call into question the reliability of basing DLOM conclusions on these small restricted stock studies.

Section 2 – Pre-IPO Studies

Pre-IPO studies analyze otherwise identical stocks of a company by comparing prices before and as-of the IPO date.[18] Even more than the restricted stock studies, the valuation utility of the pre-IPO studies is seriously flawed. For example, the “before” dates of these studies use different measurement points ranging from several days to several months prior to the IPO.[19] Determining a “before” date that avoids market bias and changes in the IPO company can be a difficult task.[20] If the “before” date is too close to the IPO date, the price might be affected by the prospects of the company’s IPO. If the “before” date is too far from the IPO date, overall market conditions or company specific conditions might have changed significantly. Such circumstances undermine the use of pre-IPO studies to estimate a specific DLOM.

The IRS DLOM Job Aid discusses three pre-IPO studies: the Willamette Management Associates studies; the Robert W. Baird & Company studies; and the Valuation Advisors’ Lack of Marketability Discount Study.[21] Each of these studies suffers from deficiencies that undermine their usefulness for estimating the DLOM applicable to a specific business as of a specific date. First, the Willamette and Baird & Company studies were of limited size and are not ongoing. The Willamette studies covered 1,007 transactions over the years 1975 through 1997 (an average of 44 transactions per year), while the Baird & Company studies covered 346 transactions over various time periods from 1981 through 2000 (an average of 17 transactions per year).[22] While the Valuation Advisors studies are ongoing and larger than the others, covering over at least 12,533 transactions from 1985 to November 2017, it represents an average of about 380 pre-IPO transactions per year.[23] Although larger than the restricted stock studies discussed in the previous section, the sample sizes of these pre-IPO studies remain small on an annual basis and

[18] Internal Revenue Service, *Discount for Lack of Marketability Job Aid for IRS Valuation Professionals,* page 19.

[19] Id.

[20] Id. page 21.

[21] Id. page 19.

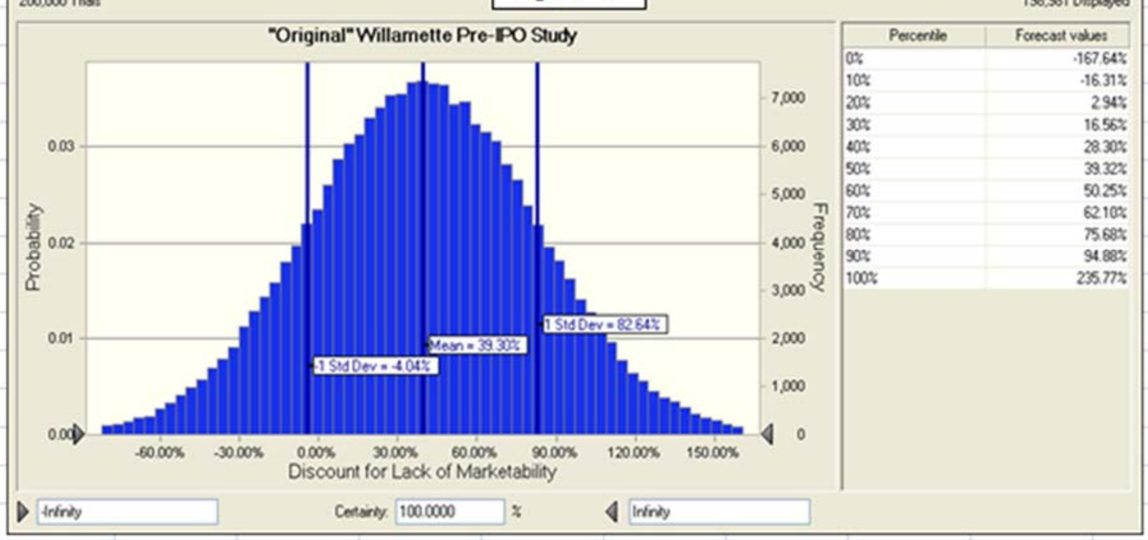
[22] Id.

[23] *See* description of the Valuation Advisors Lack of Marketability Discount Study at http://www.bvmarketdata.com/defaulttextonly.asp?f=Valuation%20Advisors%20Lack%20of%20Marketability%20Discount%20Study%20-%20DLOM%20Database%20(Discount%20for%20Lack%20of%20Marketability)

subject to considerable data variation.[24] This fact alone calls the reliability of the pre-IPO studies into question.

Second, the Willamette and Baird & Company studies report a broad range of averages, and very high standard deviations relative to their means reflecting the broad range of underlying data points.[25] The "original" Willamette studies report mean discounts that average 39.1% and standard deviations that average 43.2%.[26] The "subsequent" Willamette studies report mean discounts that average 46.7% and standard deviations that average 44.8%.[27] And the Baird & Company studies report mean discounts that average 46% and standard deviations that average 45%.[28] Figure 3.3 was prepared using *Crystal Ball* to model a 200,000-trial normal statistical distribution based on the reported means and standard deviations of the "original" Willamette studies. It discloses that a potential range of discounts comprising the 39.1% mean discount of this study extends from *negative* 167.6% to positive 235.8%.

Figure 3.3



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[24] Internal Revenue Service, *Discount for Lack of Marketability Job Aid for IRS Valuation Professionals,* page 15.

[25] The standard deviation of the Valuation Advisors study is not available on its website.

[26] Internal Revenue Service, *Discount for Lack of Marketability Job Aid for IRS Valuation Professionals,* page 95.

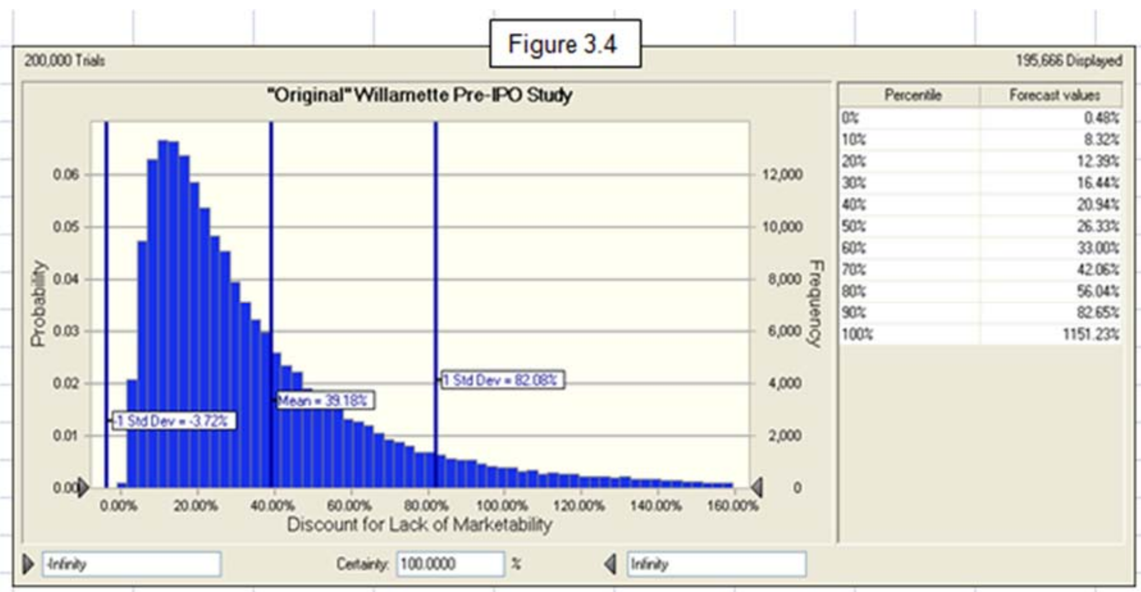
[27] Id. page 96.

[28] Id. page 97.

Applying the same normal distribution analysis to the “subsequent" Willamette studies and to the Baird & Company studies, we find that:

- The potential range of discounts comprising the “subsequent” Willamette studies is from *negative* 151.2% to positive 239.9%.
- A 206-observation subset of the aforementioned Baird & Company studies reports average mean discounts of 44% and average standard deviations of 21%.[29] The potential range of discounts comprising this study is from *negative* 59.8% to positive 150.6%.

As with the restricted stock studies, common sense tells one that a DLOM cannot be negative. Therefore, normal statistical distribution cannot be the appropriate assumption regarding the distribution of discounts within the populations for pre-IPO study discounts, and a log-normal distribution must be assumed instead. Using *Crystal Ball* with the log-normal assumption and 200,000 trials resulted in the graph below. It discloses that the log-normal range of discounts comprising the “original” Willamette study is from 0.5% to 1151.2% with a median discount of 26.3%. Almost 70% of probable outcomes occur below the 39.2% mean discount of the study.



Applying the same log-normal distribution analysis to the “subsequent" Willamette studies and to the Baird & Company studies, we find that:

- The potential range of discounts comprising the “subsequent” Willamette studies is from 1.3% to 1,192.9% with a median discount of 33.8%. Over 60% of probable outcomes occur below the mean discount of the study.

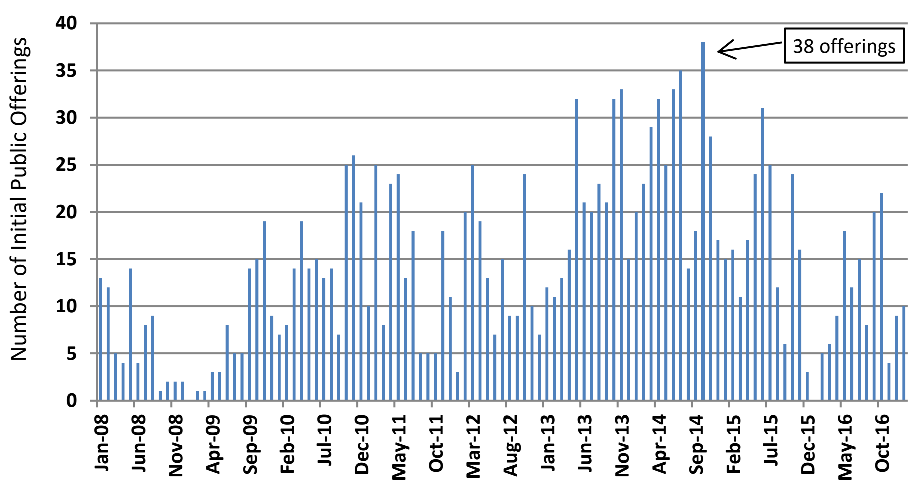
[29] Z. Christopher Mercer, Quantifying Marketability Discounts (2001), page 80.

- The potential range of discounts comprising the Baird & Company studies is from 5.7% to 327.3% with a median discount of 42.7%. Approximately 60% of probable outcomes occur below the mean discount of the study.

The discount distribution problems of the pre-IPO studies and the inability to align with (a) past and present market dynamics; (b) a specific valuation date; and (c) a specific valuation subject, seriously call into question the reliability of basing DLOM conclusions on pre-IPO studies.

Third, the volume of IPO transactions underlying the pre-IPO studies is shallow and erratic as shown in Figure 3.5.[30] In the approximately nine years ending January 2017 the peak volume of public offerings was 38 (October 2014). And in January 2009 and January 2016 there were no IPOs at all. The average number of offerings was 14.4 per month, but from September 2008 through March 2009 the average number of IPOs priced was less than 1.3 per month. It is difficult to understand a rationale for estimating DLOM for a specific privately held company at a specific point in time based on such sparse data.

Figure 3.5
Number of Initial Public Offerings Priced in the Month



Fourth, the Tax Court has found DLOMs based on the pre-IPO approach to be unreliable. The court concluded in *McCord v. Commissioner* that the pre-IPO studies may reflect more than just the availability of a ready market. Other criticisms were that the Baird & Company study is biased because it does not sufficiently take into account the highest sales prices in pre-IPO transactions and the Willamette studies provide insufficient disclosure to be useful.[31]

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[30] http://www.nasdaq.com/markets/ipos/activity.aspx?tab=pricings

[31] *McCord v. Commissioner,* 120 T.C. 358 (2003)

Chapter 4
THE PLURIS® RESTRICTED STOCK DATABASE

Many practitioners use the Pluris® DLOM database ("Pluris® database") to benchmark discounts, or use the companion calculator to compute DLOM. Pluris® states, "With this data your determination of an appropriate marketability discount for your valuation will be based on actual transaction data, not on an opinion, prior court cases, or a median value from a smaller study."[32] This chapter analyzes the reliability of benchmarking and calculating DLOMs. The analysis uses Version 4.2.0 of the Pluris® Database, which is dated November 21, 2014.

Section 1 — What Is the Pluris® DLOM Database?

The Pluris® database is a listing of restricted stock private placement transactions that is updated quarterly.[33] The source of the reported transactions is the PrivateRaise database, which, according to its website, "is the leading source for comprehensive analysis of private investments in public equity (PIPEs), Reverse Mergers, Shelf Registrations, and Special Purpose Acquisition Companies (SPACs)."[34]

The Pluris® database obtained for analysis includes 3,632 restricted stock transactions from January 2, 2001, to June 30, 2014. The transactions include issuers whose stock is or was traded on the following exchanges: NASDAQ-Capital Market (CM), NASDAQ-Global Market (GM), NASDAQ-Global Select Market (GS), NYSE, NYSE Amex, Over-the-Counter (OTC), and OTC Bulletin Board (OTC BB).

Each transaction in the Pluris® database potentially contains 76 fields of data. Not every transaction reports complete data. The basic information provided for each transaction includes:

- Issuer name
- Ticker symbol
- The primary exchange for issuer's securities
- Standard Industrial Classification ("SIC") code
- Industry sector
- Issue date
- Gross proceeds
- Common stock discount or premium

Pluris® states, "[R]estricted shares of public companies are marketable…[and] can be sold in private transactions, at a discount."[35] But what does the discount represent? Pluris® and

[32] http://www.pluris.com/pluris-dlom-database

[33] http://www.pluris.com/files/PDFs/Pluris_DLOM_flyer.pdf

[34] http://www.privateraise.com/about/about1.php

[35] Pluris® DLOM database Discussion prepared for NACVA on June 5, 2010, at slide 6.

many practitioners simply assume that restricted stock discounts equate to DLOM, but if that assumption were accurate, then a discount would be reported for each of the 3,632 transactions in the database. Instead, 443 transactions occurred at sale prices equal to or above the publicly traded stock price. Those sold at prices higher than the corresponding public market price sold for price premiums, not price discounts.

The existence of restricted stocks sold at price premiums relative to the public market price is strong evidence that factors other than DLOM affect the prices reported for restricted stocks. Consequently, there may be no reasonable basis for benchmarking DLOM against a population or sub-population of restricted stock transactions. The uncertainty of composition of restricted stock discounts is exacerbated by problems measuring the discounts in some instances (e.g., when warrants are a part of the transaction) and by the lack of correlation of the observed discounts with any of the available financial metrics. These problems are discussed in detail later in this chapter.

The restricted stock transactions that occurred at price premiums do not conform to the generally held view that marketability restrictions result in valuation discounts relative to fully liquid investments, and actually undermine the notion that restricted stocks are an appropriate benchmark for estimating DLOM. Setting that contradiction aside, including zero and negative discount transactions in DLOM estimation inappropriately shrinks the average discount. Restricted stocks sold with no discount or at a premium price (i.e., a negative discount) relative to the publicly traded price cannot represent DLOM and should be excluded from DLOM benchmarking exercises.

Importantly, the nature of the restriction(s) attached to each restricted stock listed in the Pluris® database is not disclosed. Nor are they disclosed in The Stout Study, which is discussed in Chapter 5. This negates the ability to reach an informed conclusion, solely using the data reported in the databases, regarding the extent to which the observed discount represents compensation for lack of marketability or compensation for something else. For example, the discount may simply represent price leverage possessed by a large provider of capital over an issuer who needs money, or any number of unknown causes besides a lack of marketability.

Section 2 — Are the Pluris® Transactions "Accurate"?

The analyses presented in this paper assume that data collection for the Pluris® database is reasonably accurate. Practitioners should, however, verify the accuracy of the specific transactions underlying their DLOM conclusions, and recalculate the discounts observed by Pluris®.

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http://www.pluris.com/files/PDFs/Pluris_DLOM_Database_Demo.pdf

Section 3 — Some Identified Problems with the Pluris® DLOM Calculator

A variety of other problems with the Pluris® database and calculator were identified that practitioners may need to address:

- The medians calculated using the Pluris® RSED Method 1 included with Version 4.2.0 were based on 3,450 transactions, instead of the entire population of 3,632 transactions. This resulted in 182 transactions being excluded from Pluris® DLOM calculations.[36] The analyses herein correct this omission.
- The "DownloadCalculations" tab of the Pluris® database takes data from the "Data" tab and calculates the quartile median for each of eight valuation parameters. When a transaction does not have a value for the parameter, the blanks are counted as zeros. This has the effect of miscalculating downwardly the medians of quartiles. For our analyses, the analyses herein reflect a corrected formula to exclude blank cells from the calculations of the median values. This ensures that blank cells have no impact on the DLOM calculations.
- When a transaction in the database has a value equal to the demarcation between two quartiles, the Pluris® DLOM methodology places the transaction in both quartiles. For example, when a transaction has one million dollars of total revenue for the preceding 12 months, the Pluris® methodology puts this transaction in both the third quartile (one million to nine million dollars) and the fourth quartile (zero to one million dollars). This has the obvious, but apparently minor, effect of double counting the transaction. No adjustment was made for this issue.
- Some transactions have different announcement and closing dates. For example, Solitario Exploration & Royalty Corp. (ticker: XPL) announced its transaction a week before the reported February 28, 2014, closing date.[37] The announcement caused an immediate spike in the price of Solitario's common stock resulting in an increased discount—based on the closing date—reported in the Pluris® database. There is no apparent assurance that discounts measured in such circumstances are proper. No adjustment was made for this type of defect.
- Some restricted stock transactions require the issuer to register the stocks after the closing date or else penalties are applicable. For example, Derma Sciences, Inc. (ticker: DSCI) sold common stock on November 8, 2007. DSCI was required to file a registration statement no later than January 7, 2008, and to use its best effort to cause the registration statement to be declared effective no later than March 5, 2008. Failing

[36] This is observed in the "DownloadCalculations" tab of the Pluris spreadsheet download, and may be unique to Pluris® DLOM database Version 4.2.0.

[37] Solitario Exploration & Royalty Corp. Form 8-K dated February 28, 2014.

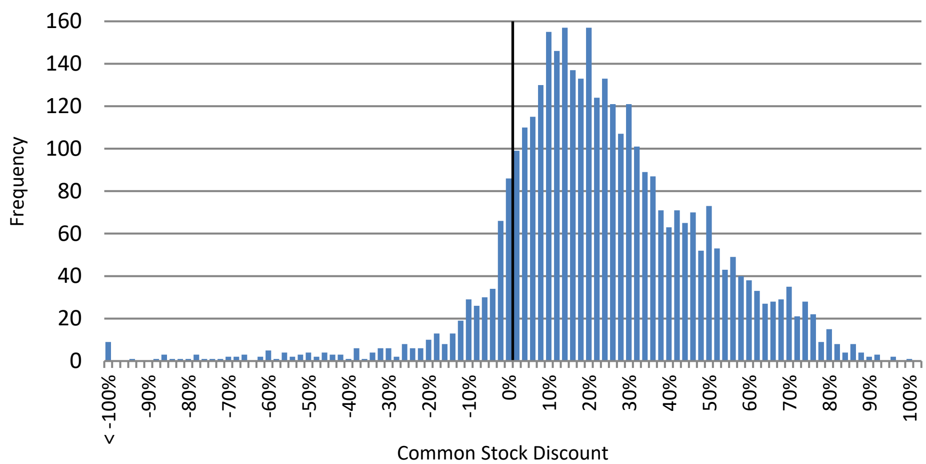
to do so would subject DSCI to a penalty.[38] The Pluris® database does not seem to account for the effects of such obligations on the transaction pricing. It is reasonable to believe that observed discounts would be greater and observed premiums lesser but for such registration obligations. No adjustment was made for this type of issue.

Section 4 — Pluris® Discount / Premium Measurement

The discounts and premiums in the Pluris® database represent the difference between the closing price of the corresponding publicly traded stock of the issuer and the calculated price per share of the restricted stock on the date of its issue. Figure 4.1 shows the distribution of the reported discounts for the 3,632 restricted stock transactions in Version 4.2.0 of the database. As can be readily seen, many of the reported discounts are zero or negative. Negative discounts (i.e., restricted stocks sold at premium prices) are inconsistent with DLOM concepts, and support the notion that the observations may not reflect true discounts for lack of marketability.

Figure 4.1

Distribution of 3,632 Restricted Stock Discounts in the Pluris® DLOM Database



Unlike the transactions used in other restricted stock discount studies or databases, the Pluris® database includes transactions that have warrants attached to them.[39] Pluris® estimates the fair market value of the warrants and removes that value from the gross proceeds of the transaction.[40] The intended result is the common stock portion of the transaction proceeds.[41]

[38] Derma Sciences, Inc. Form 8-K dated November 8, 2007.

[39] See Pluris® DLOM database and http://www.pluris.com/DLOM-database-construction.

[40] http://www.pluris.com/DLOM-database-construction

[41] *Ibid.*

Instead of using Black-Scholes or other option models, Pluris® uses its LiquiStat™ data to determine the value of restricted stock private placement transactions with warrants.[42] Pluris® states that it is its opinion that Black-Scholes and other theoretical models overvalue warrants.[43]

Of the 3,632 transactions in the Pluris® database, 1,867 had warrants attached, representing 51% of the transactions in the database. Of the 3,189 transactions reporting discounts greater than zero, 1,760 (55%) had warrants attached. Table 4.1 shows that there is a material difference in average restricted stock discounts depending on whether warrants attach to the transactions.

Table 4.1

	All Transactions		With Warrants		Without Warrants	
Restricted Stock Discount	Count	Average Discount	Count	Average Discount	Count	Average Discount
All transactions	3,632	22.4%	1,867	30.3%	1,765	14.0%
Discounts > Zero	3,189	28.4%	1,760	33.6%	1,429	22.0%

The average discount for the 3,632 restricted stock transactions comprising the entire Pluris® database is 22.4%. Reducing the population to the 3,189 transactions with reported discounts that are greater than zero increased the average discount to 28.4%. Further investigation revealed that the reported discounts for transactions involving warrants are dramatically greater than for transactions without warrants. Looking only at the transactions with reported discounts greater than zero, we found that those with warrants reported an average discount of 33.6% compared to 22.0% for the transactions without warrants—53% more discount.

The dichotomy of warrant and warrantless transactions is an example of how other factors can affect the amount of the discount. For example, LiquiStat may undervalue the portion of the transaction value attributable to the warrants, thereby inflating the supposed restricted stock discount;[44] or transactions with warrants may represent riskier stocks or other factors that require greater compensation for the investor; or perhaps warrants should be considered valueless, which would further increase the restricted stock discount; or perhaps the warrant has a much higher value than Pluris® estimated, reducing the restricted stock discount. Furthermore,

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[42] *Ibid.*

[43] *Ibid.*

[44] Undervaluing warrants decreases the portion of the discount attributed to the warrant while increasing the portion attributed to the restricted stock, thereby increasing the reported restricted stock discount.

the discrepancy in discounts between transactions with and without warrants may differ from industry-to-industry, time, and other factors. It would be prudent for practitioners to investigate the value of any warrants attached to restricted stock transactions before relying on them or the Pluris® discount percentages. Alternatively, it may be prudent to exclude transactions with warrants from one's DLOM analysis. It is beyond the scope of this practice aid to test values that Pluris® assigned to warrants; the reported Pluris® discounts are therefore taken at face value for the analyses described in this paper.

Section 5 — Discount Correlation with Total Assets, Market Value-to-Book Value Ratio, 12-Month Stock Price Volatility, Percentage of Shares Outstanding, and Calendar Quarters to Sell

A June 5, 2010, presentation[45] by Pluris® to the National Association of Certified Valuators and Analysts (NACVA) included several graphs (shown below) that suggest a strong association between the restricted stock discounts reported in the Pluris® database and certain underlying transactional metrics. Those metrics are total assets; market value-to-book value ratio; 12-month stock price volatility for the publicly traded stock; block size of the restricted stock as a percentage of shares outstanding; and block size (quarters to sell). The strength of the associations is tested in the subsections below after grouping the discounts reported in the Pluris® database into size-based deciles—the same method employed by Pluris®.

Section 5.A Discounts and Block Size

The presumption is that block size is an indicator of the illiquidity of the shares sold in the private placement because the largest blocks are very hard to trade out of after the placement and would tend to be viewed by most buyers as more akin to private equity than public equity that is temporarily restricted.[46] The presumption can be tested by comparing discounts and the percentage of shares outstanding represented by a block of stock. Figure 4.2 shows that a strong R-square of correlation results when the discounts of the restricted stock transactions reported in the database are grouped into size-based deciles according to the percentage of shares outstanding.

The positive association between percentage-block size and restricted stock discount is somewhat illogical. Considering that the companies comprising the Pluris® database are publicly traded, one could reasonably conclude that corporate control increases as the percentage of outstanding shares represented by the block size increases. Given that controlling interests are considered to be easier to sell than minority interests, the relationship of observed discounts to block size should be negative, not positive.

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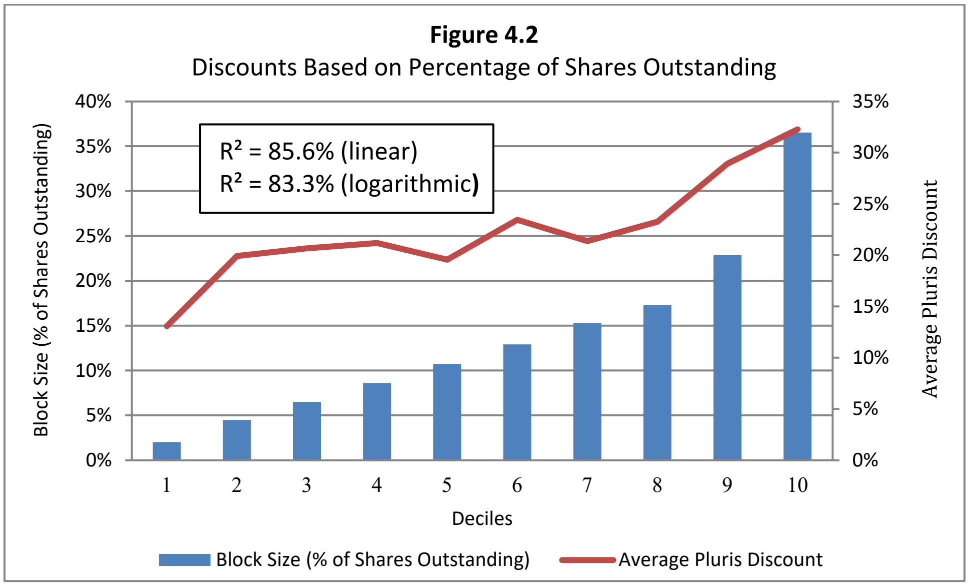
[46] http://www.pluris.com/files/PDFs/Pluris_DLOM_Database_Demo.pdf, slide 18.



Applying the regression formulas to assumptions of 1% and 100% blocks of shares outstanding results in implied discounts from 9.1% to 63.7% depending on the regression formula used as per Table 4.2. It is incongruous that the projected discount range is so different between the two formulas considering that the R-squares of correlation of the linear and logarithmic regressions are so closely aligned. It would be difficult, if not impossible, to state whether a 100% owned private company should have a 35% or a 64% DLOM based solely on this information.

Contrary to the presumption underlying the block size analysis is the fact that the Pluris® database includes 517 transactions with stock discounts of 50% or greater for which the average percentage block size is just 18% of shares outstanding. This fact, lost by grouping the data in deciles, suggests that discounts do not increase with block size.

Table 4.2

Implied Range of Discounts Based on Pluris® Deciles of Blocks of Stock as a Percentage of Shares Outstanding

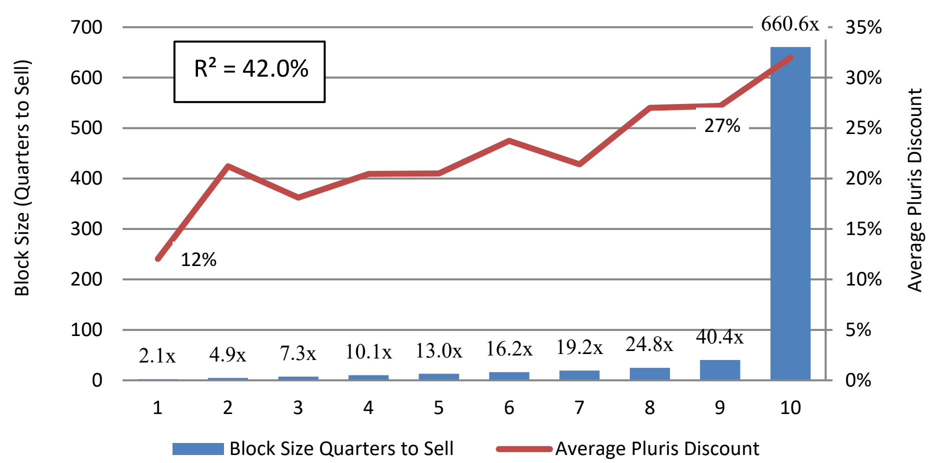
	1% of Shares Outstanding	100% of Shares Outstanding
Linear Regression y = 0.4787x + 0.158	16.3%	63.7%
Logarithmic Regression y = 0.0574ln(x) + 0.3532	9.1%	35.3%

The block size presumption can also be tested by grouping the discounts of the reported restricted stock transactions according to the number of calendar quarters required to sell the block of stock under the SEC Rule 144 "dribble out" provision. This comparison is complicated by the fact that the number of quarters reported is very high for many of the transactions in the

Pluris® database. Figure 4.3A shows that according to Pluris® the average 10th decile transaction requires 660 quarters to liquidate the position. Moreover, the discounts associated with the transactions with much longer purported periods of illiquidity seem illogically low. An asset that cannot be sold for years should have a 100% DLOM at some point. The fact that the average discount of the ninth and 10th decile transactions does not approach 100% is an indication that the discounts do not exclusively reflect illiquidity. Or it may be that the presumption that discounts are associated with the dribble out provision is defective because it ignores that a sale of the entire block could be made in a private transaction in a potentially much shorter period of time. It is therefore not unreasonable to conclude that SEC Rule 144 dribble out time periods are not appropriate indicators of DLOM for blocks of restricted stocks that will require long time periods for selling into the public markets absent some provision that prohibits or limits sale of the block in non-public transactions.

Figure 4.3A indicates a weak 42.0% linear R-square of correlation between deciles of reported discounts and Rule 144-based time periods. But when the data is considered logarithmically, as in Figure 4.3B, the analysis indicates a strong 82.1% R-square of correlation—virtually double the linear regression result.

Figure 4.3A
Discounts Based on Quarters to Sell



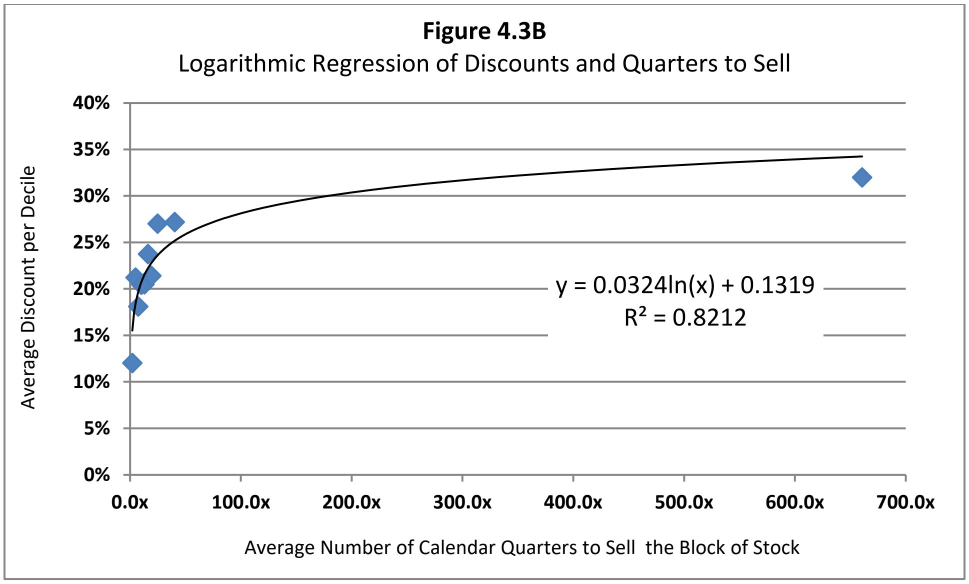


Table 4.3 shows the range of discounts implied by applying the logarithmic regression formula to time periods equal to 1 calendar quarter and 660 calendar quarters. It is illogical that the discount for a required holding period of one calendar quarter should be about 13% while the discount for a required holding period of 165 *years* should be just 34%.

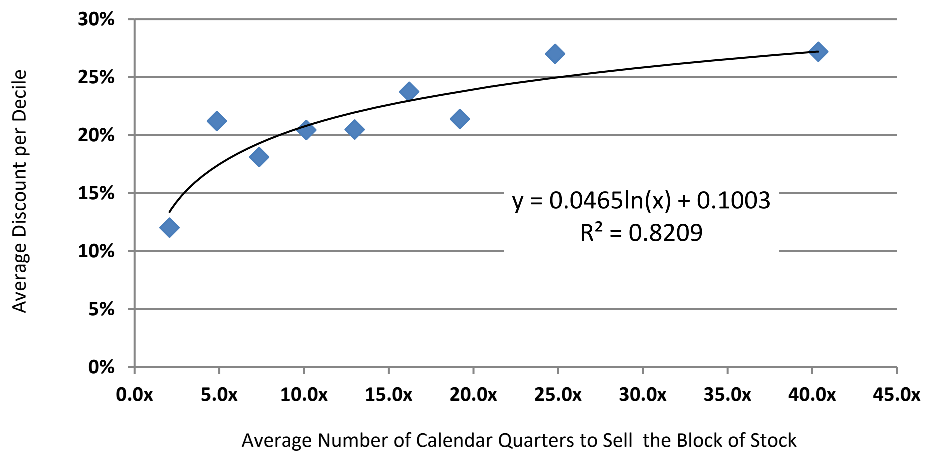
Table 4.3

Implied Range of Discounts Based on Pluris® Deciles of the Number of Calendar Quarters Needed to Sell a Block of Stock Applying Rule 144

	1 Quarter to Sell	660 Quarters to Sell
Logarithmic Regression y = 0.0324ln(x) + 0.1319	13.2%	34.2%

Figures 4.3A and 4.3B display obvious extreme skewing of the 10th decile time periods. Figure 4.3C shows that excluding the 10th decile makes only a negligible change the R-square—three hundredths of a percent. The R-square of correlation per Figure 4.3B is 82.12% while per Figure 4.3C it is 82.09%.

Figure 4.3C
Logarithmic Regression of Discounts and Quarters to Sell without the 10th Decile



Excluding the 10th decile of transactions changes the regression formula in a way that results in the broader range of predicted results shown in Table 4.4 compared to Table 4.3. Whether the 10th decile transactions are included or excluded, however, one must question basing DLOM conclusions on the metric of Rule 144 calendar quarters. It seems unreasonable to conclude that a holding period of 165 years would result in a discount of just 34% to 40%. Something much closer to 100% seems appropriate.

Table 4.4

Implied Range of Discounts Based on Nine Pluris® Deciles of the Number of Calendar Quarters Needed to Sell a Block of Stock Applying Rule 144 (Omits the 10th Decile)

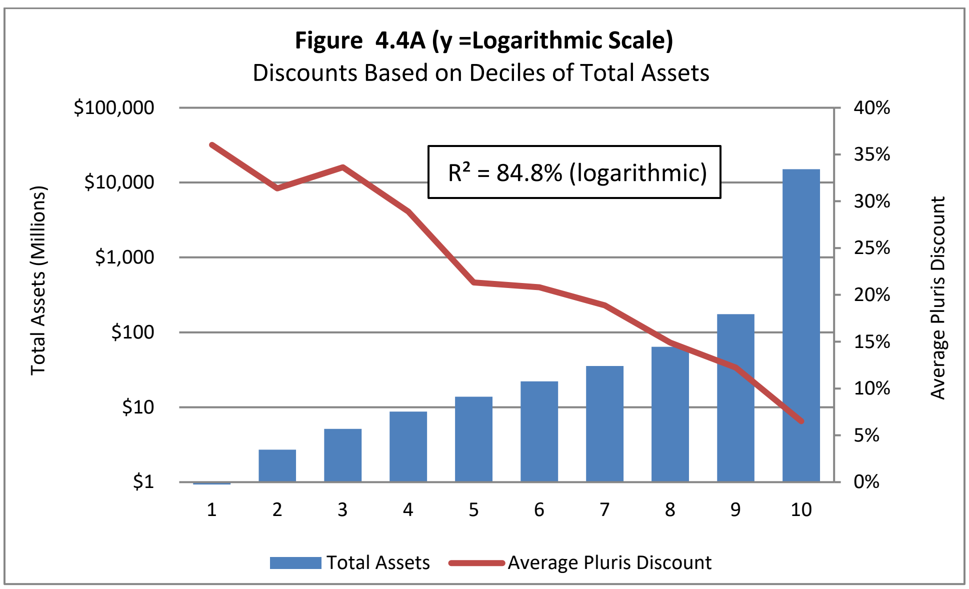
	1 Quarter to Sell	660 Quarters to Sell
Logarithmic Regression $y = 0.0465\ln(x) + 0.1003$	10.0%	40.2%

The above analyses of discounts based on decile groupings of the percent of shares outstanding seem to support a presumption that block size is an indicator of the illiquidity of the shares sold in the private placement, despite being somewhat illogical. But the increasing discounts associated with increasing percentage-block sizes could easily be attributable to additional or alternative causes. Possible explanations of the discounts may be investor negotiating strength as the size of the block increases, issuer compulsion, other forms of compensation granted by the issuer to the investor, the industry of the issuer, or other factors. Practitioners would be right to question the reasonableness of basing discount and DLOM conclusions on the percentage of shares outstanding that is represented by a block of stock because of the possible logical disconnection.

Section 5.B Firm Size Comparisons

Many investors will accept the premise that the size of an enterprise is an indicator of its riskiness and investment attractiveness. Risk is believed to be lower for larger firms. Correspondingly, the presumption is that smaller companies require significantly deeper discounts than larger companies. Indeed, Figure 4.4A bears this presumption out. It shows that a strong 84.8% logarithmic R-square of correlation results when the discounts of the restricted stock transactions reported in the Pluris® database are grouped into size-based deciles according to total assets.

Figure 4.4A shows that the 10th decile companies are very large. Omitting the 10th decile from the analysis increased the R-square of correlation 92.1%. However, reliability requires correlations to be reasonably consistent across the full body of data that includes the 10th decile, so the approach of excluding the 10th decile simply because the asset values are large is rejected.[47] Nevertheless, the reported total assets seem to provide a strong explanation of the variation in the observed discounts, which tentatively supports using total assets to predict discounts.



[47] The R-square of correlation using linear regression is a weak 33.5%. Omitting the 10th decile transactions are omitted from the analysis increased the R-square of linear correlation to 59.7%. However, as stated, reliability requires correlations to be reasonably consistent across the full body of data without arbitrary exclusions, so the approach of excluding the 10th decile is again rejected.

Figure 4.4B presents the logarithmic regression of the data shown in Figure 4.4A. Note that the relationship is virtually a straight line declining from approximately 33% discount for the smallest companies to 0% discount for the largest companies. Table 4.5 presents the range of discounts predicted by the regression formula shown in Figure 4.4B.

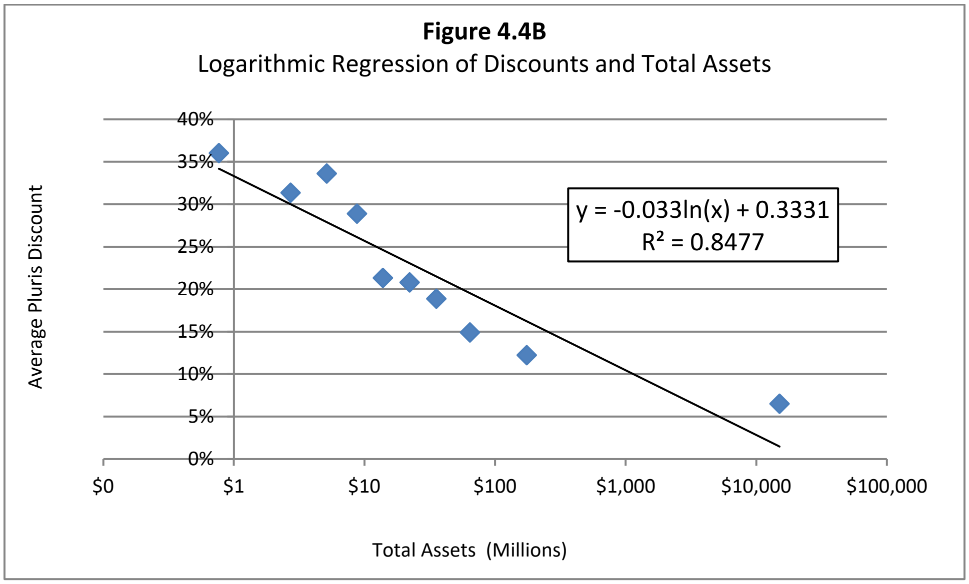


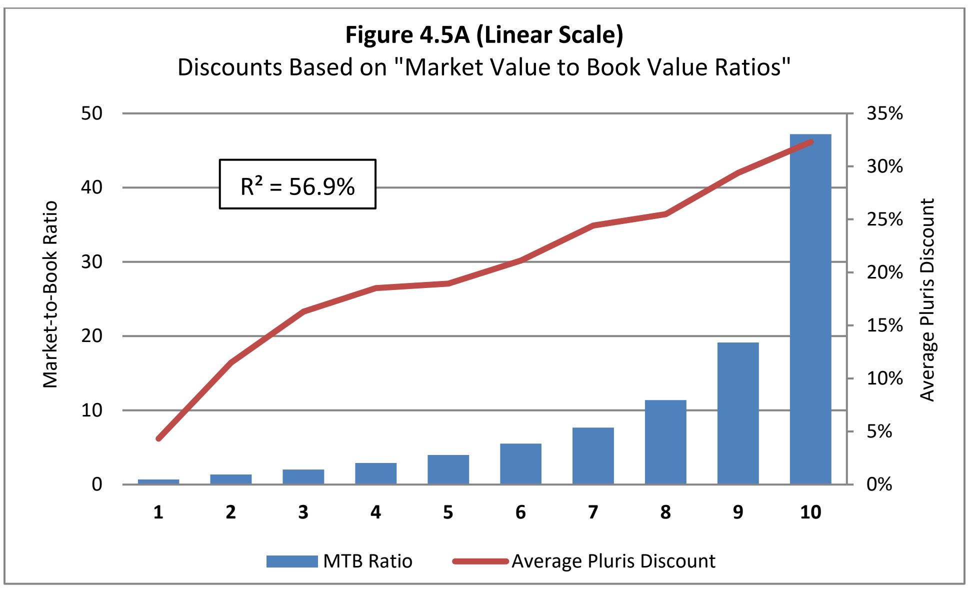
Table 4.5
Implied Range of Discounts Based on Pluris® Deciles Total Assets

	$100,000 of Total Assets	$24.2 Billion of Total Assets
Logarithmic Regression $y = -0.033\ln(x) + 0.3331$	40.9%	0.0%

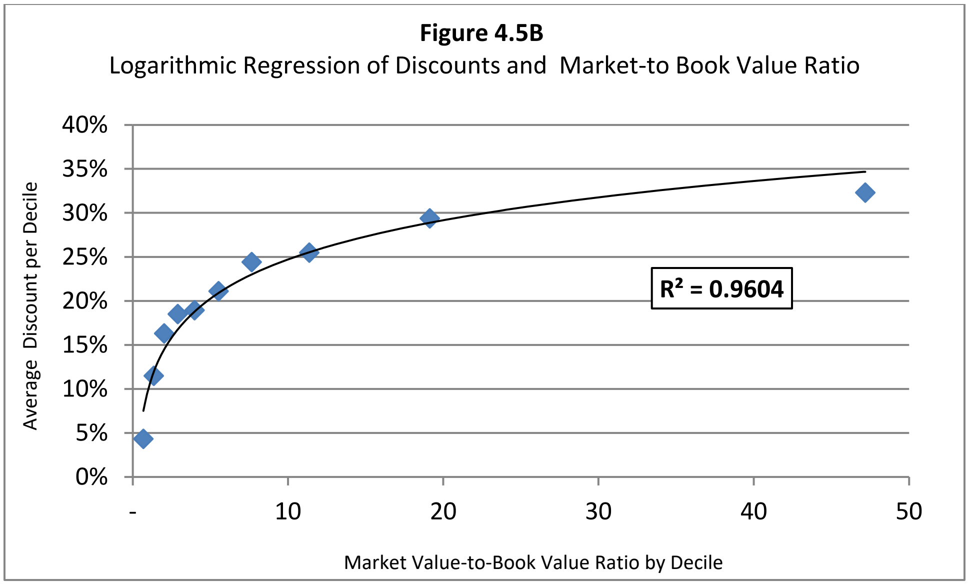
Many practitioners may find a 40% DLOM implied for very small companies to be reasonable. But what if the assets of the small firm is cash or some other highly liquid asset? Or what if the assets are time shares? Should the DLOM still be 40% under those circumstances? Logic suggests otherwise. Likewise, the zero DLOM implied for very large companies suggests that there should be little or no price risk associated with an illiquid marketing period. The volatility of the stock market—even for very large companies—contradicts this notion. These considerations undermine the use of discounts based on total assets as a benchmark for DLOM estimation. Nevertheless, logarithmic analysis of discounts on the decile groupings of total assets supports a conclusion that size is an indicator of riskiness and investment attractiveness.

Section 5.C Balance Sheet Strength

Many investors will accept the premise that the balance sheet strength of an enterprise is an indicator of its riskiness and investment attractiveness. Issuers with weaker balance sheets presumably require deeper discounts. Some investors associate the ratio of market value-to-book value with balance sheet strength, and assume that higher multiples are a sign of financial weakness. But the ratios do not necessarily describe the financial strength of enterprises' highly appreciated asset values. Mining, real estate-heavy, and many technology companies come to mind in this regard. Instead of being a sign of balance sheet weakness, market value-to-book value ratios can be a sign of strength to the extent that the ratio is a measure of investors' expectations regarding the effectiveness with which a business can generate profits from its assets. In such situations higher ratios are an indication of superior performance expectations (financial growth) that come from stronger business plans, more valuable assets, stronger market positions, stronger management, and/or other strength factors. Nevertheless, market value-to-book value ratios appear to be a predictor of the restricted stock discounts reported in the Pluris® database. Grouping the transaction discounts reported in the database into deciles according ranking by market value-to-book value ratio yields a modest R-square of correlation of 56.9% as Figure 4.5A shows.



Although it is intuitive that balance sheet strength is an indicator of riskiness and investment attractiveness, and that issuers with weaker balance sheets require significantly deeper discounts, a 56.9% R-square of correlation leaves a lot of "noise" unaccounted for. Logarithmic regression provides a better answer to the question, resulting in the very high 96.0% R-square of correlation shown in Figure 4.5B.



We can conclude from Figure 4.5B that (1) market value-to-book value ratios are a strong predictor of restricted stock discounts (regardless of whether the ratio indicates balance sheet "strength," and (2) the relationship between market value-to-book value and restricted stock discounts is logarithmic and not linear. But the regression line of Figure 4.5B implies a ceiling for discounts despite no apparent ceiling on the market value-to-book value ratio. Table 4.6 shows that the implied discount for a company with a market value-to-book value ratio of 1000:1 is 54.3%. This seems unreasonably low given a premise that such a ratio represents a very financially weak enterprise.

Table 4.6

Implied Range of Discounts Based on Pluris® Deciles of Market-to-Book Value Ratio

	1x Ratio	1000x Ratio
Logarithmic Regression y = 0.0643ln(x) + 0.0991	9.9%	54.3%

Section 5.D Stock Price Volatility

Stock price volatility is a direct measure of the risk associated with a stock. One would accordingly expect a strong association between the price volatility of the publicly traded stock of a restricted stock issuer and the negotiated transaction discounts. This expectation was tested by creating decile groupings of the discounts reported in the Pluris® database according to the reported 12-month price volatilities. Figure 4.6A shows a 58.4% R-square of correlation using linear regression that, as with the market value-to-book value ratio linear regression analysis, leaves a lot of "noise" unaccounted for.

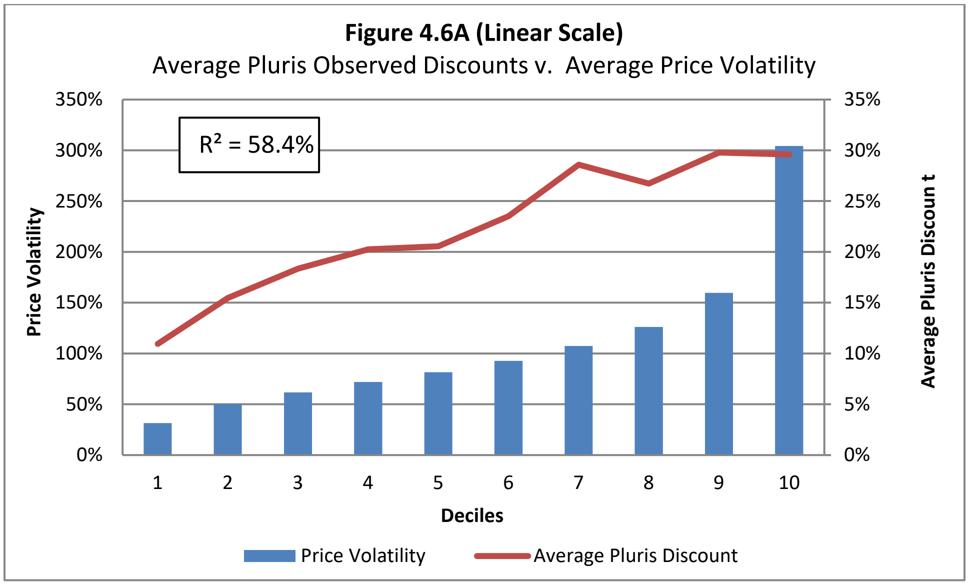
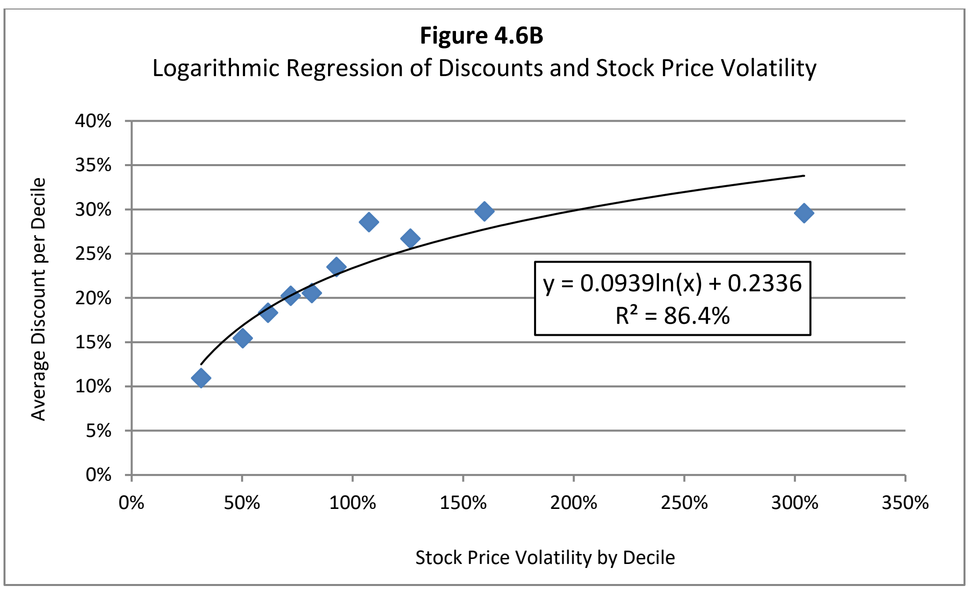


Figure 4.6B considers the same data using logarithmic regression. Again, as with the market value-to-book value ratio analysis, the statistical relationship improves dramatically—to an 86.4% R-square of correlation in this instance. Based on the decile analysis at least, stock price volatility appears to strongly influence restricted stock discounts.



We can conclude from Figure 4.6B that (1) stock price volatilities are a strong predictor of restricted stock discounts, and (2) the relationship between price volatility and restricted stock discounts is logarithmic and not linear. But the regression line of Figure 4.6B implies a ceiling for discounts that seems inconsistent with very high price volatility situations. Table 4.7 shows that the implied discount for a company with a price volatility of 1000% is just 45.0%. This again seems unreasonably low.

Perhaps not illogically, the price volatility regression formula implies a price premium for very low price volatility stocks. Table 4.7 shows an implied 19.9% *premium* for companies with 1% price volatility.

Table 4.7
Implied Range of Discounts Based on Pluris® Deciles of Price Volatility

	1% Price Volatility	1000% Price Volatility
Logarithmic Regression y = 0.0939ln(x) + 0.2336	(19.9)%	45.0%

Section 5.E Testing Statistical Significance

The correlations of block size (percentage of shares outstanding and quarters to sell), total assets, market-to-book value, and annual stock price volatility were tested for statistical significance. This was a two-part test. The first part involved identifying the 2,496 restricted stock transactions in the Pluris® database for which positive values are reported for all five of the metrics. The second part involved identifying the 1,162-transaction subset of the 2,496-transaction group that does not have warrants attached.

Table 4.8 shows that four of the five metrics passed the significance test; only the total assets metric failed. The variable that was most strongly significant is stock price volatility, which suggests that the restricted stock price negotiators were particularly sensitive to the price risks associated with the stocks. The market-to-book value ratio was the second strongest variable, which is not surprising considering that stocks with high ratios have substantial "blue sky," which often adds to stock price volatility. Importantly, this analysis contradicts the assumption that DLOM conclusions should be based on the total assets of a business.

Table 4.8

	2,496 Transactions Positive Value Variables				1,162 Transactions Positive Values without Warrants			
Independent Variable	R Square	*t Stat*	*P-value*	Significant?	R Square	*t Stat*	*P-value*	Significant?
Block Size (Shares Outstanding)	4.3%	10.57024	0.00000	Yes	6.4%	8.91435	0.00000	Yes
Block Size (Quarters to Sell)	0.3%	2.84111	0.00453	Yes	1.0%	3.35573	0.00082	Yes
Total Assets (Latest Quarter)	0.1%	-1.76321	0.07799	No	0.1%	-1.31879	0.18750	No
Market to Book Ratio	8.7%	15.42419	0.00000	Yes	11.2%	12.08104	0.00000	Yes
Annual Stock Price Volatility	25.0%	22.17967	0.00000	Yes	23.8%	19.03871	0.00000	Yes

Section 6 — The Discounts Reported in the Pluris® DLOM Database™ Are Not Consistent with Past Changes in SEC Rule 144 Required Holding Periods

As discussed in Section 5, the "quarters to sell" metric reported in the Pluris® database offers a statistically significant explanation of the changes in the observed discounts. Consideration is now given to whether changes in the SEC Rule 144 required holding period offers an additional explanation of restricted stock transaction discounts reported in the database. The original Rule 144 required that restricted stocks be held for two years past the issue date before they could be "dribbled out" into the public marketplace. The SEC changed the required holding period to one year effective April 29, 1997, and to six months effective February 15, 2008.[48] None of the transactions in Version 4.2.0 of the Pluris® database predates April 29, 1997, so the large holding period change from two years to one year cannot explain any of the changes in the reported restricted stock discounts.

Conventional wisdom, supported by several restricted stock studies, is that smaller restricted stock discounts have resulted from the successive changes in Rule 144 holding period.[49] A reduction in discount is logical when the restriction goes from two years to one year to six months, because the restricted block of stock can be liquidated quicker. The restricted stock studies by Columbia Financial Advisors confirmed this market reaction. *See* Table 3.1 in Chapter 3. Quickness of sale (i.e., increased liquidity) evidently reduces holding period risk.

Table 4.9 summarizes the transactions reported in the Pluris® database based on issue dates before and after the SEC's November 15, 2007, announcement date of the change from a one-year to a six-month required holding period. The announcement date is the appropriate demarcation because the rule change was applicable to stocks acquired both before and after the February 15, 2008, effective date.[50] It is assumed that negotiators of a restricted stock transaction would have known of the rule change upon the announcement by the SEC.

Table 4.9

				Transactions with Discounts Greater than Zero			
		All 3,632 Transactions		3,189 Transactions		1,429 Transactions without Warrants	
Issue Date	Restriction Period	Count	Discount	Count	Discount	Count	Discount
1/2/2001 – 11/14/2007	1 Year	2,379	23.0%	2,160	27.6%	936	21.7%
11/15/2007 – 6/30/2014	6 Months	1,253	21.2%	1,029	30.0%	493	22.5%

[48] "Discount for Lack of Marketability Job Aid for IRS Valuation Professionals – September 25, 2009," p 15.

[49] *Ibid*, p 17.

[50] https://www.sec.gov/rules/final/2007/33-8869.pdf

Table 4.9 demonstrates that the average observed discount for all transactions in the Pluris® database decreased slightly from 23.0% to 21.2% upon the announcement that the Rule 144 holding period would change from one year to six months. But the opposite is observed when only transactions with positive discounts (i.e., common stock discounts greater than zero) are included. For the latter set, the average discounts reported by Pluris® increased from 27.6% to 30.0% for one-year and six-month restriction periods, respectively. And the same occurred for the subset of transactions that did not include warrants. The average discount for warrantless transactions increased from 21.7% before the announcement date to 22.5% after the announcement date. These results are contrary to expectations.

The restricted stock discounts observed in the Pluris® database do not seem to behave consistently with the conventional wisdom regarding the effect of Rule 144 time restrictions. Therefore, one must consider that one or more things other than Rule 144 holding periods explain the discounts reported in the database. A reasonable explanation is that the negotiators of restricted stock transactions of the size comprising the database anticipate block sales instead of dribble-out sales of their holdings.

Section 7 — Correlation of the Pluris® Restricted Stock Discounts Valuation Metrics

Pluris® database reports 76 fields of data for each restricted stock transaction. Sixty-two of those fields might be considered valuation metrics. Linear regression was initially used to calculate the R-squares of correlation and regression formula slopes for these 62 metrics relative to the discounts reported by Pluris®. The results are presented in Table 4.10. The lines presented in red are the metrics used in the Pluris® DLOM calculator, except for price volatility.

Table 4.10 shows that none of the metrics exhibits a large R-square of correlation with transaction discounts, and few exhibit a regression line slope that is not essentially flat. Flat regression lines offer no predictive power.

The largest correlations occurred with the transactions for which price volatility was reported and that have discounts greater than zero. These R-squares of correlation range from 8.9% to 12.4%, and have very shallow, but positive, regression line slopes that range from 0.0588:1 to 0.0879:1. The best performing metric in the linear regression analysis was 12-month daily price volatility, with an R-square of 12.4%. This metric explains about an eighth of the variation in discounts reported in the database. Obviously, practitioners would prefer to see R-squares that are closer to 100%.

It seems unreasonable to benchmark a restricted stock DLOM on even the strongest correlations reported in Table 4.10. Benchmarking DLOMs on the even more poorly correlated parameters used in the Pluris® DLOM calculator seems problematic.

Table 4.10

Linear Regressions of Pluris® DLOM Database Financial Metrics with the Observed Discounts

	All 3,632 Transactions		3,189 Transactions with Discounts Greater than Zero	
	R^2	Slope	R^2	Slope
Price Volatility Data				
Daily volatility over 12 months	2.7%	0.0470	12.4%	0.0641
Daily volatility over 12 months or applicable period prior to issue date	3.0%	0.0445	12.4%	0.0588
Daily volatility over six months	2.1%	0.0450	12.0%	0.0709
Weekly volatility over 12 months	3.5%	0.0753	11.9%	0.0864
Weekly volatility over six months	2.9%	0.0695	11.2%	0.0879
Daily volatility over three months	2.1%	0.0435	11.0%	0.0662
Weekly volatility over three months	2.7%	0.0560	8.9%	0.0653
Price Data				
Effective purchase price per share	3.1%	(0.0056)	10.2%	(0.0067)
Purchase price	2.9%	(0.0054)	9.9%	(0.0066)
Closing price on announcement date (A+0)	2.1%	(0.0047)	9.2%	(0.0067)
Closing price three days after announcement date (A+3)	2.1%	(0.0047)	9.2%	(0.0066)
Exercise Price	2.6%	(0.0102)	8.3%	(0.0132)
Closing price 10 days after issue date (C+10)	1.4%	(0.0035)	7.2%	(0.0050)
Closing price seven days after issue date (C+7)	1.5%	(0.0035)	7.1%	(0.0050)
Closing price seven days prior to issue date (C-7)	1.5%	(0.0035)	7.1%	(0.0050)
Volume-weighted average price 10 days prior to issue date (VWAP)	1.4%	(0.0035)	7.1%	(0.0050)
Closing price three days after issue date (C+3)	1.4%	(0.0034)	7.0%	(0.0050)
Closing price one day after issue date (C+1)	1.4%	(0.0034)	6.8%	(0.0049)
Closing price one day prior to issue date (C-1)	1.3%	(0.0034)	6.8%	(0.0049)
Closing price on the issue date (C+0)	1.3%	(0.0033)	6.7%	(0.0049)
Restricted Stock Data				
Fair market value per warrant	1.6%	(0.0254)	6.7%	(0.0377)
Number of warrants	1.9%	0.0000	6.3%	0.0000
Block size (shares outstanding)	2.4%	0.4692	3.9%	0.3929
Shares sold	0.9%	0.0000	3.2%	0.0000
Gross proceeds	0.2%	(0.0000)	0.8%	(0.0000)
Common stock portion of proceeds	0.3%	(0.0000)	0.8%	(0.0000)
Warrants portion of gross proceeds	0.4%	0.0000	0.0%	0.0000
Block Size (Volume)	0.0%	(0.0000)	0.3%	0.0000
Block size (quarters to sell)	0.0%	(0.0000)	0.4%	0.0000
Issue date	0.0%	0.0000	0.5%	0.0000
Placement ID number	0.0%	(0.0000)	0.4%	0.0000
Financial and Market Data				
Market-to-book ratio	4.6%	0.0040	8.7%	0.0036
Price divided by book value per share (P/BV)	4.6%	0.0040	8.7%	0.0036
Enterprise value divided by revenue for last 12 months (EV/Revenue)	1.4%	0.0017	3.0%	0.0016
Dividend yield	0.9%	(1.6625)	2.2%	(1.9705)
Net profit margin for last 12 months	1.0%	(0.0995)	2.0%	(0.0718)

Revenue growth in 12 months prior to most recent 10-Q	0.3%	(0.0345)	1.4%	(0.0468)
Price divided by average earnings per share for last 12 months (P/E)	0.1%	(0.0003)	1.4%	(0.0007)
Shares outstanding	0.0%	0.0000	1.0%	0.0000
Market capitalization	0.2%	(0.0000)	0.8%	(0.0000)
Price divided by the estimated average earnings per share for the next two years (P/E FY+2)	1.3%	0.0009	0.6%	0.0005
Total equity on most recent 10-Q	0.2%	(0.0000)	0.5%	(0.0000)
Total revenues 12 months prior to most recent 10-Q	0.2%	(0.0000)	0.5%	(0.0000)
Beta relative to S&P 500 for 12 months prior to issue date	0.2%	(0.0001)	0.5%	(0.0001)
Trading volume over twelve months or applicable period	0.9%	(0.0000)	0.4%	(0.0000)
Trading volume multiplied by the issuer's closing stock price on issue date	0.1%	(0.0000)	0.4%	(0.0000)
Average daily trading volume over 12 months prior to issue date	0.8%	(0.0000)	0.4%	(0.0000)
Expiration Date	0.2%	0.0000	0.2%	0.0000
Average daily trading volume over six months prior to issue date	1.1%	(0.0000)	0.2%	(0.0000)
VIX as of the issue date	0.7%	(0.3533)	0.2%	(0.1261)
Z-score	0.1%	(0.0000)	0.2%	(0.0000)
Enterprise value	0.0%	(0.0000)	0.2%	(0.0000)
SEC Filing Data				
Total debt on most recent 10-Q	0.0%	(0.0000)	0.1%	(0.0000)
Average daily trading volume over three months prior to issue date	1.0%	(0.0000)	0.1%	(0.0000)
Total assets on most recent 10-Q	0.0%	(0.0000)	0.1%	(0.0000)
Average daily trading volume over one month prior to issue date	0.7%	(0.0000)	0.1%	(0.0000)
Total liabilities on most recent 10-Q	0.0%	(0.0000)	0.1%	(0.0000)
EBITDA 12 months prior to most recent 10-Q	0.0%	(0.0000)	0.1%	(0.0000)
Average daily trading volume over seven days prior to issue date	0.1%	(0.0000)	0.1%	(0.0000)
Price divided by the estimated average earnings per share for the next year (P/E FY+1)	0.2%	0.0003	0.0%	0.0001
Pre-tax income 12 months prior to most recent 10-Q	0.0%	(0.0000)	0.0%	(0.0000)
Net income 12 months prior to most recent 10-Q	0.0%	(0.0000)	0.0%	(0.0000)

Using the transactions reported in the Pluris® database, Table 4.11 summarizes the average (mean), standard deviation, and the coefficient of variation for each of eight valuation parameters comprising the Pluris® DLOM calculator. All of the metrics except net profit margin and market value-to-book value ratio exhibit very large coefficients of variation, ranging as high as 36.2:1 for total assets. In comparison, the coefficients of variation shown for restricted stock discounts are 0.7:1. The high coefficients of variation of most of the Pluris® DLOM calculator parameters undermine their predictive utility for DLOM estimation. Net profit margin (1.9 coefficient of variation) and market value-to-book value ratio (1.4 coefficient of variation) appear to be more meaningful for DLOM estimation. Twelve-month price volatility, with a coefficient of variation of .9 to 1 most closely approximates the data distribution of the restricted stock discounts. This is consistent with the higher logarithmic R-squares of correlation that price volatility exhibits with restricted stock discounts as shown in Table 4.11.

Table 4.11

Financial Parameters Used in the Pluris® DLOM Calculation Methodology

	All 3,632 Transactions			3,189 Transactions with Discounts > Zero		
	Average	Standard Deviation	Coefficient of Variation	Average	Standard Deviation	Coefficient of Variation
Pluris® DLOM Calculator Parameters						
Total Assets	$1,535,520,816	$55,612,171,960	36.2	$1,658,322,427	$59,281,730,272	35.7
Revenues	194,563,230	2,152,944,264	11.1	190,652,639	2,252,841,333	11.8
EBITDA	33,622,178	1,138,721,628	33.9	34,695,161	1,210,638,417	34.9
Net Income	(9,182,288)	135,947,965	14.8	(9,006,978)	140,872,735	15.6
Net Profit Margin	-16.7%	31.4%	1.9	-17.4%	31.8%	1.8
Total Equity	99,717,239	1,156,263,919	11.6	96,301,461	1,208,685,423	12.6
Enterprise Value	805,238,151	19,858,204,899	24.7	854,743,284	21,150,553,918	24.7
Market-to-Book Ratio	10.2	14.7	1.4	10.5	14.9	1.4
Comparative Parameters						
12-Month Daily Price Volatility	124.2%	116.7%	.9	124.5%	119.0%	1.0
Restricted Stock Discounts	22.4%	29.9%	.7	28.4%	19.9%	.7

Table 4.12 summarizes three transactional subsets of the Pluris® calculator metrics that were analyzed using both linear and logarithmic regression. All of the subsets were limited to those with positive discounts. Further limitations imposed were (1) transactions with stated metric values (that is, not zero or blank); (2) transactions with only positive metric values; and (3) transactions without warrants. Out of 3,632 transactions in the database, only 840 transactions satisfied the first limitation, only 323 transactions satisfied the first two limitations, and only 253 transactions satisfied all three limitations. Logarithmic regression significantly improved the regression results within the 840-transaction subset vis-à-vis linear regression, with the exception of net profit margin subset. The 6.81% R-square of correlation for that metric is considered an anomaly created by the presence of 517 transactions with negative net profit margins.

Table 4.12

R-Squares of Correlation of Positive Discount Transaction Subsets of the Pluris® Database

	Transactions With and Without Warrants				Transactions Without Warrants	
	840 with Non-Zero Metrics		323 with Positive Metrics		253 with Positive Metrics	
	Linear	Logarithmic	Linear	Logarithmic	Linear	Logarithmic
Total Assets	0.26%	17.58%	0.42%	12.62%	0.41%	7.19%
Revenues	0.57%	10.32%	0.47%	4.09%	0.29%	0.74%
EBITDA	0.12%	Neg. Values	0.01%	6.64%	0.04%	2.78%
Net Income	0.24%	Neg. Values	0.33%	5.15%	0.11%	2.39%
Net Profit Margin	6.81%	Neg. Values	0.68%	0.29%	1.80%	1.80%
Total Equity	0.80%	16.34%	0.86%	10.84%	0.64%	6.68%
Enterprise Value	0.32%	11.83%	0.45%	7.03%	0.40%	2.98%
Market-to-Book Ratio	3.10%	5.99%	2.02%	6.91%	2.72%	6.98%
12-month daily volatility	16.07%	19.01%	21.34%	20.30%	26.72%	20.52%

Table 4.12 confirms that the restricted stock discounts reported in the database correlate poorly on a linear basis with the parameters employed in the Pluris® calculator even with refined data subsets. But that is not the case with price volatility, for which the refined subsets yield linear and logarithmic R-squares of correlation improve with the dataset refinement.

Table 4.13 shows the R-squares of correlation of the Pluris® calculator parameters with the restricted stock discounts reported in the Pluris® database. Explanatory power increases somewhat as the group membership is narrowed from the 840 to 253. this is demonstrated by the R-squares of correlation increasing from 20.89 to 29.1%. This may mean that DLOM conclusions should not be based on parameters with negative values and that transactions with warrants should be ignored.

Finally, the statistical significance of the Pluris® calculator parameters was considered for the Table 4.12 subgroups. Table 4.13 shows that only price volatility is statistically significant across all three subgroups, with t-Stats greater than +/- 2 and p-Values less than 0.05. The net profit margin metric is statistically significant for the 840 and 253-transaction groups. The market value-to-book value metric is statistically significant for the 840-transaction group. None of the other Pluris® calculator parameters are statistically significant for any of the three subgroups of transactions.

Table 4.13
Statistical Significance of Positive Discount Transaction Subsets of the Pluris® Database

	Transactions With and Without Warrants				Transactions Without Warrants	
	840 with Non-Zero Metrics		323 with Positive Metrics		253 with Positive Metrics	
	R-Square = 20.8%		R-Square = 23.0%		R-Square = 29.1%	
	t Stat	*P-value*	*t Stat*	*P-value*	*t Stat*	*P-value*
Total Assets	1.339047	0.180922	0.699754	0.484601	0.481411	0.630658
Revenues	-1.124144	0.261277	-0.909178	0.363956	-0.927224	0.354730
EBITDA	1.389704	0.164991	0.668319	0.504422	0.439243	0.660876
Net Income	1.601365	0.109677	1.169352	0.243152	1.391995	0.165197
Net Profit Margin	-4.207082	0.000029	-1.341499	0.180731	-2.161333	0.031647
Total Equity	-0.947467	0.343677	-0.991707	0.322106	-0.821767	0.412015
Enterprise Value	-1.198804	0.230946	-0.583396	0.560047	-0.410644	0.681695
Market-to-Book Ratio	3.544787	0.000415	1.041203	0.298585	0.997442	0.319542
12-month daily volatility	10.513029	0.000000	8.155209	0.000000	8.278852	0.000000

The above analyses provide a reasonable basis to conclude that negotiators of restricted stock transactions are much more concerned with the price volatility of the investment than with other valuation metrics when pricing their transactions.

Section 8 — Using the Pluris® Database for Benchmarking

The limited number of transactions in the Pluris® database makes the identification of appropriate benchmarks unlikely for most valuations. Although the database consists of 3,362 restricted stock transactions, there are only 2,085 unique issuers. Even the larger number is likely to be insufficient for reasonable benchmarking. The transactions comprising the database stretch over a period beginning January 2, 2001, and ending June 30, 2014—a period of 4,928 days. On average, there is less than one transaction per database day. Even if the number of days is reduced to the approximate number of stock-trading days during the database period (about 3,375 days), there are only 1.1 transactions per day on average.

Table 4.14 shows the percentage distribution of transactions in the Pluris® database. About 58% of the potential valuation days have no transactional data. Table 4.14 also shows that about 38% of all trading days have no transactional data.

Table 4.14

Number of Pluris® Transaction Occurrences by Day

Number of Occurrences	All Database Days: Number of Days	All Database Days: Percent	Stock Trading Days: Number of Days	Stock Trading Days: Percent
0	2,843	57.69%	1,290	38.23%
1	1,151	23.36%	1,151	34.10%
2	554	11.24%	554	16.41%
3	245	4.97%	245	7.26%
4	77	1.56%	77	2.28%
5	34	0.70%	34	1.00%
6	12	0.24%	12	0.36%
7	9	0.18%	9	0.27%
8	2	0.04%	2	0.06%
9	1	0.02%	1	0.03%
Total Days	4,928	100.00%	3,375	100.00%

Figure 4.7 shows the number of transactions in the database that occurred on the same day. One day—December 30, 2005—had nine stock transactions. Two days had eight transactions. Nine days had seven transactions. And so forth. No transactions are reported for 2,843 days in the 2001 to 2014 data range.

Figure 4.7
Number of Pluris Restricted Stock Transactions that Occurred on the Same Day
(from January 1, 2001 to June 27, 2014 - 4,926 total days)

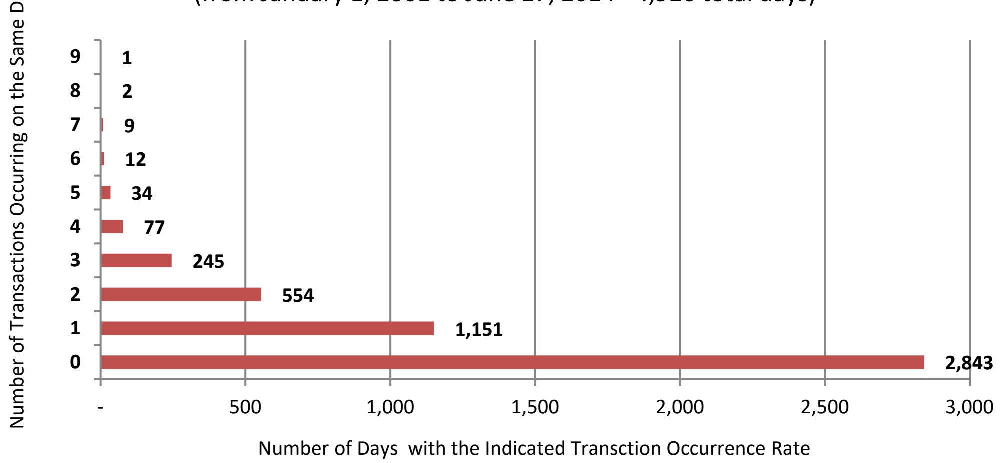
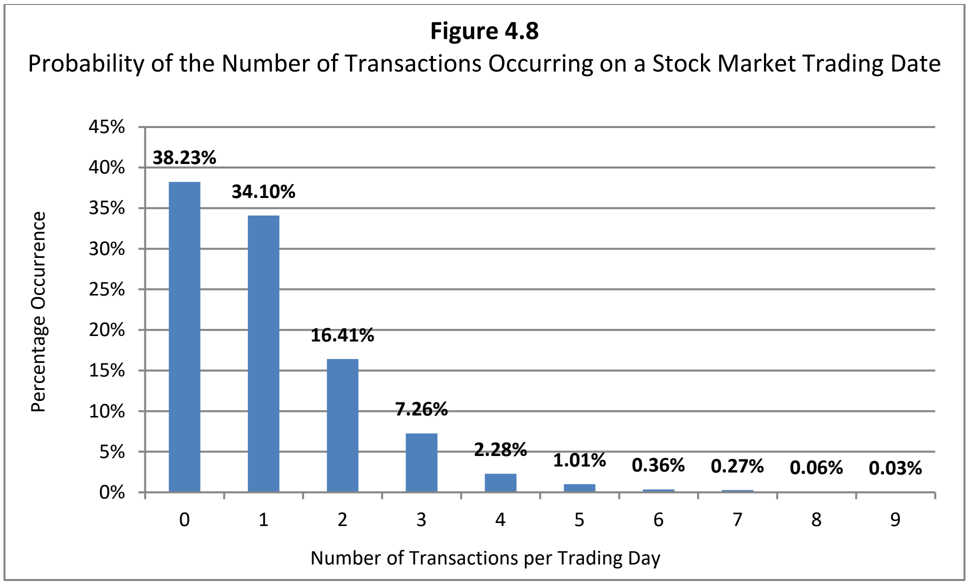


Figure 4.8 presents the distribution of transactions based on stock market trading days shown in Table 4.14.



One problem represented by a small volume of transaction data is the likelihood that none or only one transaction is reported for a particular valuation date. Another problem is the reporting lag that occurs as the database is compiled. Before a transaction can be added to the database, it must be reported by the issuer and then picked up by Pluris®. This imposes a limitation on data availability for contemporary valuations. But it may be that practitioners can compensate for the problems of too-limited of transaction data by selecting transactions that occurred over a longer period than the one day represented by the valuation date. Of course, the longer the time span required to capture the number of transactions that a practitioner may deem sufficient, the more risk there is that economic circumstances may have materially changed during the span.

Lack of comparability of the reported transactions to a valuation subject is another problem associated with too little transactional data. The fewer the number of transactions, the more work the analyst must likely do to establish comparability.

Moreover, it is problematic that nothing is known about how the transaction negotiators actually priced their restricted stocks. Accordingly, there is a significant risk of appraisers failing to consider the same things considered by the stock negotiators. Unlike the public marketplace for which it can be presumed that investors' decisions are based on publicly available information, restricted stock pricing is a matter of private negotiation. It is speculation to assume that their motivations align with any particular piece of information selected for benchmarking. It is further problematic that narrowing transaction selection to seek comparability inhibits the ability to find any potentially relevant transaction to use as a benchmark for a valuation subject.

The 3,632 transactions in the Pluris® database cover 377 four-digit SIC codes. However, the official SIC system identifies at least 1,005 four-digit industries.[51] The database therefore covers only 37.5% of the potential industry classifications. The database is also highly concentrated. More than half of all transactions in the database are comprised of just 17 four-digit codes—less than two percent of the possible classifications. These 17 are shown in Table 4.15. Furthermore, more than 25% of all transactions in the database fall into just four SIC codes: 2834, 7372, 8731, and 1311. If a practitioner is valuing a business that operates in one of these four industries, then the chance of finding one or more reasonably comparable transactions is enhanced. Identifying a satisfactory comparable transaction involving one of the 13 other SIC codes comprising the group of 17 most frequently occurring codes would be much more difficult. The difficulty finding a comparable transaction is an order of magnitude greater if the valuation subject's industry is not one of the 17 most frequently occurring SIC codes.

Table 4.15

Distribution of Pluris® Transactions by 4-Digit SIC Code

Industry	SIC Code	Frequency of Occurrence	Percent of Pluris® Transactions
Pharmaceutical Preparations	2834	368	10.1%
Prepackaged Software	7372	232	6.4%
Commercial Physical and Biological Research	8731	187	5.1%
Crude Petroleum and Natural Gas	1311	143	3.9%
Subtotal			25.5%
Surgical and Medical Instruments and Apparatus	3841	113	3.1%
Gold Ores	1041	94	2.6%
State Commercial Banks	6022	94	2.6%
Oil and Gas Field Exploration Services	1382	76	2.1%
Information Retrieval Services	7375	73	2.0%
Biological Products, except Diagnostic Substances	2836	68	1.9%
Electromedical and Electrotherapeutic Apparatus	3845	67	1.8%
Business Services, not elsewhere classified	7389	65	1.8%
Semiconductors and Related Devices	3674	64	1.8%
Computer Integrated Systems Design	7373	59	1.6%
In Vitro and In Vivo Diagnostic Substances	2835	51	1.4%
National Commercial Banks	6021	42	1.2%
Telephone Communications, except Radiotelephone	4813	37	1.0%
The Above 17 Four-Digit SIC Codes		1,833	50.5%
The 360 Other Four-Digit SIC Codes[52]		1,799	49.5%
Total Pluris® Database		3,632	100.0%

[51] http://siccode.com/en/pages/what-is-a-sic-code

[52] Only 137 of these industry codes have at least four transactions, which is the minimum necessary to satisfy a quartile methodology such as Pluris® uses in its DLOM calculator.

The 17 four-digit codes represent 1,833 restricted stock transactions, ranging in count from 368 transactions involving the pharmaceutical preparations industry to 37 transactions involving the non-radio telephone communications industry. Industries falling outside of the top-17 are represented by just five transactions on average; and 119 transactions have unique SIC codes within the database. Coupled with the fact that the database covers a 13.5-year period, it may be impossible to identify time-relevant transactions in the four-digit industry classification of a valuation subject. Adding other elements of comparability to the benchmarking process greatly exacerbates the task.

When practitioners are faced with an impossible task at an initially desired level of detail, they often zoom out their analyses to base conclusions on a broader analytical view. With that in mind the industry distribution of the Pluris® database was analyzed on a two-digit SIC code basis.[53] *See* Table 4.16. Just eight broad industries account for 2,546 (over 70%) of the 3,632 transactions in the database. The remaining 1,086 transactions are spread over 61 broad industries—an average of less than 18 transactions per industry, and about one per nine months of time range in the database.

Table 4.16
Distribution of Pluris® Transactions by 2-Digit SIC Code

Industry	SIC Code	Occurrences	Percent of Transactions
Chemicals and Allied Products	28	591	16.3%
Business Services	73	574	15.8%
Measuring, Analyzing, And Controlling Instruments	38	298	8.2%
Oil and Gas Extraction	13	262	7.2%
Electrical and Other Electrical Equipment and Components except Computer Equipment	36	235	6.5%
Engineering, Accounting, Research, Management, and Related Services	87	214	5.9%
Metal Mining	10	198	5.5%
Depository Institutions	60	174	4.8%
The Above 8 Two-Digit SIC Codes		2,546	70.1%
The 61 Other Two-Digit SIC Codes		1,086	29.9%
Total Pluris® Database		3,632	100.0%

Assuming a practitioner wanted to identify a single benchmark transaction from the four-digit SIC code with the most transactions (Pharmaceutical Preparations, code 2834) for a single

[53] As of this writing, there are 83 two-digit codes in the SIC system. Fourteen SIC codes are not represented in Version 4.2.0 of the Pluris® DLOM database.

generic database day, the probability of a successful outcome would be about 4.3%.[54] This, of course assumes an even spread of the database transactions over time. Assuming the practitioner wanted to benchmark against a single generic stock trading day, the probability of finding a benchmark would increase to about 6.2%.[55] This is because there are only about 3,375 stock trading days over the 13.5-year time period of the database, and assumes that all database transactions are spread evenly over time and occurred on stock trading days. Assuming, however, that two or more transactions in that four-digit industry classification were desired, the 6.2% probability would fall to about 2.8%.[56] This is because about 72.3% of potential stock trading days have zero or only one transaction occurrence. *See* Table 4.14. Obviously, the fewer transactions there are for a particular industry in the database, and the more daily occurrences desired by a practitioner, the less likely it is that a potentially comparable transaction will be found.

The problem of lack of comparability becomes much more acute if a practitioner sets out to benchmark against a particular company within an industry. Few companies within an industry issue restricted stocks, and those that do may not be comparable to other industry participants (including the valuation subject). Substantial professional judgment is required to justify comparability.

Examining the stock issuers that comprise the Pluris® database reveals a total of 2,085 unique issuers, of which 1,271 (61%) have one restricted stock transaction reported. These single-transaction issuers account for 35% of the transactions in the database. The other 814 issuers (39%) have multiple stock transactions reported, accounting for 2,261 of the stock transactions in the database. Thus, 39% of the issuers account for 65% of the transactions reported in the database, which give these issuers a dramatically disproportionate influence on reported values. Table 4.17 summarizes the occurrences of restricted stock transactions by stock issuer.

[54] That is 10.1% per Table 4.15 times 42.31% of the database days that have at least one transaction occurrence per Table 4.14.

[55] That is 10.1% per Table 4.15 times 61.75% of the database stock trading days that have at least one transaction occurrence per Table 4.14.

[56] That is 10.1% per Table 4.15 times 27.67% of the database stock trading days that have at least two transaction occurrences per Table 4.14.

Table 4.17
Frequency of Restricted Stock Transactions by Individual Issuers

Frequency of Transactions	Issuers Number	Issuers Percentage	Transactions Number	Transactions Percentage
15	1	0.0%	15	0.4%
11	1	0.0%	11	0.3%
10	1	0.0%	10	0.3%
9	3	0.1%	27	0.7%
8	6	0.3%	48	1.3%
7	10	0.5%	70	1.9%
6	27	1.3%	162	4.5%
5	44	2.1%	220	6.1%
4	88	4.2%	352	9.7%
3	180	8.6%	540	14.9%
2	453	21.7%	906	24.9%
	814	39.0%	2,361	65.0%
1	1,271	61.0%	1,271	35.0%
	2,085	100.0%	3,632	100.0%

The limited number of issuers and disproportionate number of stock issues per issuer pose two problems for practitioners. First, transaction concentration among a subset of issuers makes the likelihood of matching a preferred issuer with a particular valuation date even more remote than discussed above. Second, the average of the discounts observed by Pluris® is disproportionately influenced by a few active restricted stock issuers. These circumstances undermine the usefulness of basing DLOM conclusions directly on benchmarked transactions in the Pluris® database.

Section 9 — Using the Pluris® Methodology for Calculating DLOM

The Pluris® DLOM methodology involves calculating two values: Restricted Stock Equivalent Discount (RSED) and Private Equity Discount Increment (PEDI). According to Pluris®, the RSED represents an illiquid position that does not directly relate to the lack of marketability of a privately held business. But while restricted stock discounts represent illiquid positions, they do not necessarily represent DLOM. The RSED calculated by Pluris® may overstate DLOM by including non-DLOM compensation to the investor, or may understate DLOM as shown by the Pluris® database transactions with negative discounts. Regardless, the restricted stocks comprising the Pluris® database represent shares in companies with publicly traded classes of stock. That is not the situation with the stock of a privately held business, which should be more illiquid and require a greater discount than the pure DLOM associated with the restricted stock of a publicly traded company. Pluris® developed its PEDI concept to account for this difference in circumstances.[57] It is this author's view that the addition of a PEDI to a DLOM based on a publicly traded benchmark is a reasonable thing to do.

Section 9.A How Pluris® Calculates its Restricted Stock Equivalent Discount (RSED)

Pluris® calculates the RSED component of a DLOM using two methods. Method 1 is called, "Analysis of Data Download." Method 2 is called, "Analysis of Entire Database."

- For Method 1—Analysis of Data Download. The user searches and sorts the transactions in the Pluris® database to identify transactions considered comparable to the valuation subject. Transactions may be filtered by SIC code, industry sector, price volatility, market capitalization, revenue, assets, block size, or many other criteria. The calculation of RSED is then based on the filtered transactions.[58] As discussed in Section VI, finding reasonably comparable transactions within the database is highly unlikely.

- For Method 2—Analysis of Entire Database. It is generally the case that all of the restricted stock transactions are included in the calculation of RSED. The exceptions are: (1) if the time between the issue date and first announcement date exceeds 90 days; (2) if the daily stock trading volume of the publicly traded issuer is less than $5,000; (3) if the publicly traded stock of the issuer is a penny stock (i.e., ten-day volume-weighted average price prior to issue date is less than one dollar); and (4) if the publicly traded stock of the issuer trades on the pink sheet market.[59]

Once the transaction pool is established, Methods 1 and 2 calculate RSED the same way. The Pluris® calculator uses eight specified valuation parameters to calculate RSED: total assets, total revenues, EBITDA, net income, net profit margin (calculated automatically when the user enters amounts for net income and total revenue), equity (book value), enterprise value, and the ratio of market value-to-book value.[60] For each parameter, the calculator then divides the Method 1 or Method 2 transaction selections into quartiles. For each quartile, the median common stock discount is identified. Accordingly, each quartile of each parameter ultimately

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[57] "Pluris DLOM Database Discussion" prepared for NACVA, June 5, 2010, at slide 5.

[58] DLOM Database Webinar at https://www.youtube.com/watch?v=ndkowdw2aBU.

[59] See the discussion at the top of the "DLOMCALCULATION" tab of the Pluris® spreadsheet download. The number of exclusions that occur from these limitations was not determined.

[60] This is observed in the "RSED" tab of the Pluris spreadsheet download. The calculator also offers two input fields to accept parameters defined by the user. The custom fields are ignored in this discussion of the calculator methodology.

represents a single data point for which fifty percent of all transactions in the quartile have larger values, while fifty percent have smaller values.

The user enters the valuation parameters for the subject company into the RSED calculation spreadsheet, which returns the median restricted stock discount for the quartile in which the subject company input falls. Calculated RSED percentages are, therefore, based generally on just eight median data points—one for each of the specified parameters—extracted from either: (a) the specifically selected at least one transaction for Method 1; or (b) the 3,632 (before limitation) restricted stock transactions comprising the transaction database for Method 2. The Pluris® calculator then averages the median values for each parameter that the user put into the calculator to arrive at the RSED percentage.

Additional problems with the Pluris® DLOM methodology will be discussed later, but one evident at this stage is that 87.5% of all transaction values are greater than the first quartile medians and 87.5% of all transaction values are less than the fourth quartile medians of each of the calculation parameters. Relative to the second quartile medians, 62.5% of all transactions have greater values, and relative to the third quartile medians, 62.5% of all transactions have smaller values for each of the calculation parameters. These facts leave a great many values omitted from resulting DLOMs, even if one is only concerned with the eight parameters specifically comprising the calculator. Importantly, a Pluris® RSED can never be less than the median value of the lowest quartile nor more than the median value of the highest quartile. This artificial bracketing magnifies the unreliability of the low correlation of the financial parameters and observed discounts from which RSED values are calculated. It seems a problematic methodology that generally relies on just eight median data points out of the more than 220,000 pieces of data available for the 3,632 transactions in the database. For the eight specific parameters of the calculator, there are 29,048 available pieces of data that are essentially ignored.[61]

Section 9.B How Pluris® Calculates its Private Equity Discount Increment (PEDI)

Pluris® calculates PEDI by comparing the discounts associated with the largest block transactions (presumed to be the least liquid) to the average and median discount indications of all the transactions in the database.[62] Pluris® says, “Underlying this methodology is the notion that transactions in the largest blocks serve as the best proxy for the lack of marketability of small blocks of stock in privately held companies,” and, therefore, a PEDI should be added to the RSED when the subject company is not a public company issuing restricted stock. But, the

[61] That is, 3,632 transactions x 8 metrics = 29,056 – 8 medians = 29,048 transactions whose sole purpose is to establish a median.

[62] “Pluris DLOM Database Discussion” prepared for NACVA, June 5, 2010, at slide 5.

largest blocks of stock may not be the least liquid. In the context of restricted stocks of publicly traded companies, larger blocks might represent greater control. Conventional wisdom is that liquidity increases as control increases. Institutional investors may actually prefer larger blocks to smaller blocks of restricted stocks, which the SEC Rule 144 discussion of Section 6 suggests. And it likely is more economical to register large blocks than small blocks of restricted stock, which would enhance liquidity and lower discounts. The possible investor preference for large blocks of restricted stocks contradicts the method by which Pluris® measures PEDI.

Regardless of the validity of the Pluris® association of large blocks of restricted stock with small blocks of ownership in privately held businesses, making an addition to RSED such as adding a PEDI seems appropriate. The PEDI addition recognizes that the RSED calculation is based on public company transactions, and that there is a lesser marketability of interests in privately held businesses.

Figure 2.1 discussed in Chapter 2 presents a relational stratification of the types of empirical studies that researchers have performed to explore the cost of illiquidity. Figure 2.1 presents the studies in relative position based on marketing time and price volatility—assuming all other characteristics of the investment are identical. The underlying assumption is that as investments in otherwise identical companies become more illiquid and decrease in control, they become riskier. That is, they take longer to sell and are subject to greater price volatility. The PEDI concept is consistent with the Figure 2.1 depiction of incremental levels of discount that increase with sale/marketing-restriction periods and stock price volatility. Figure 2.1 demonstrates that PEDIs should increase as stocks become less liquid and/or are subject to greater price volatility relative to publicly traded stocks generally and relative to restricted stock transactions such as those comprising the Pluris® database.

Private sales of companies for which hypothetical buyers and sellers have no expectation of going public should be worth less than the restricted stock of an otherwise identical company that is public or has the anticipation of an IPO event. A PEDI is therefore an appropriate component of the DLOM applicable to an ownership interest in a private company.

Non-controlling interests in private companies require even greater discounts because the risks associated with lacking control cause the periods of time needed to liquidate the position to be potentially much longer than for the controlling interest in the same company. The Pluris® PEDI does not distinguish between controlling and non-controlling interests in privately held businesses.

As previously described, Pluris® calculates PEDI as the difference in the discounts observed for larger block and smaller block transactions.[63] The first step in this process is to compare the RSED of the largest block transactions with the average and median RSED

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[63] "Pluris DLOM Database Discussion" prepared for NACVA, June 5, 2010, at slide 5.

indications for the entire database.[64] The Pluris® calculator then uses a matrix that combines multiplicative and additive adjustments to the RSED to arrive at PEDI estimates.[65] The matrix calculates four values, but the Pluris® calculator only averages the two middle values when computing a PEDI.[66] Therefore, the Pluris® methodology ignores the possibility that the appropriate PEDI may be less than or greater than the average of the middle range of values.

Pluris® claims that the excess discount of the largest restricted stock transactions (the least liquid blocks according to Pluris®) over that of the population of transactions is analogous to the difference in discounts between public companies and private companies.[67] This analogy seems illogical as it is generally accepted that the DLOM applicable to a controlling interest should be less than that applicable to a non-controlling interest in the same company. Some practitioners argue that controlling interest investments should have no DLOM at all. Therefore, the largest blocks of restricted stocks would not be expected to be less liquid than the smallest blocks.

Additional problems with the way Pluris® measures PEDI are that it presents no evidence that: (a) it actually takes longer to sell a larger than a smaller block of a public company's restricted stock in a single private transaction; (b) there is greater price volatility exposure for larger blocks than smaller blocks of a public company's restricted stock; or (c) size-percentage blocks of public companies' restricted stocks are analogous to any particular size-percentage interests in privately held businesses. Therefore, although the concept of a PEDI is appropriate (i.e., an incremental discount relative to public company restricted stock discounts), the Pluris® logic behind the methodology seems speculative and unreliable.

Section 9.C The Pluris® Quartile Approach Creates Artificial DLOM Values

Pluris® states that its purpose is providing users with a "determination of an appropriate marketability discount…based on actual transaction data, not on an opinion, prior court cases, or a median value from a smaller study."[68] The Pluris® methodology contradicts this purpose by calculating DLOM from its own, albeit larger, "study."

DLOMs calculated using the Pluris® methodologies do not directly reflect the observed discounts for the user-selected valuation parameters (i.e., a selected combination of the total assets, total revenues, EBITDA, net income, net profit margin, book equity, enterprise value, and

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[64] *Ibid.*

[65] *Ibid*.

[66] This is observed in the "PEDI" tab of the Pluris spreadsheet download.

[67] Pluris® DLOM database Webinar at https://www.youtube.com/watch?v=ndkowdw2aBU.

[68] http://www.pluris.com/pluris-dlom-database.

the ratio of market value-to-book value). Instead, they reflect the median discount of the quartile of transaction data in which each parameter falls, thereby ignoring the range and distribution of discounts within each quartile, and explicitly ignoring all of the other available financial metrics (again, recognizing that the user can employ up to two custom parameter fields). The medians are then averaged. This method imposes an artificial floor and ceiling for the potential DLOM outcomes, as shown in Table 4.18 and Figures 4.9 and 4.10.

Pluris® Method 1 and Method 2 DLOMs were calculated for the 3,581 restricted stock transactions in the database that had complete data for all eight of the standard input parameters of the Pluris® database.[69] Transaction selection for Method 1 was the entire database. Although Method 1 is intended for selection of specific transactions that the user considers comparable to the valuation subject, that concept is illogical if the valuation subject is the very transaction being used as the benchmark. Thus, our calculations of Methods 1 and 2 DLOMs merely measure the effects of the mathematical limitations that Pluris® imposes on the Method 2 dataset. Table 4.18 shows that the difference is slight.

Table 4.18 summarizes the minimum, average, and maximum values for RSED, PEDI, and DLOM using Pluris® RSED Methods 1 and 2 as described in the previous paragraph.

Table 4.18

Pluris® Average DLOMs for 3,581 Restricted Stock Transactions

	Method 1			Method 2		
	RSED	PEDI	DLOM	RSED	PEDI	DLOM
Minimum	9.9%	5.2%	15.1%	9.4%	5.1%	14.5%
Average	21.2%	7.2%	28.4%	20.4%	7.1%	27.5%
Maximum	34.9%	9.6%	44.5%	32.3%	9.1%	41.4%

Figures 4.9 and 4.10 are histograms that show the frequency of occurrence of DLOMs calculated using RSED Methods 1 and 2, respectively.

[69] Fifty-one of the 3,632 transactions in Version 4.2.0 of the Pluris® DLOM database do not have data for the valuation parameters used by the Pluris calculator. We excluded these transactions from the calculations summarized in Table 4.17.

Figure 4.9
Distribution of Discounts and Pluris DLOMs for 3,581 Restricted Stock Transactions
Calculated using Pluris Method 1

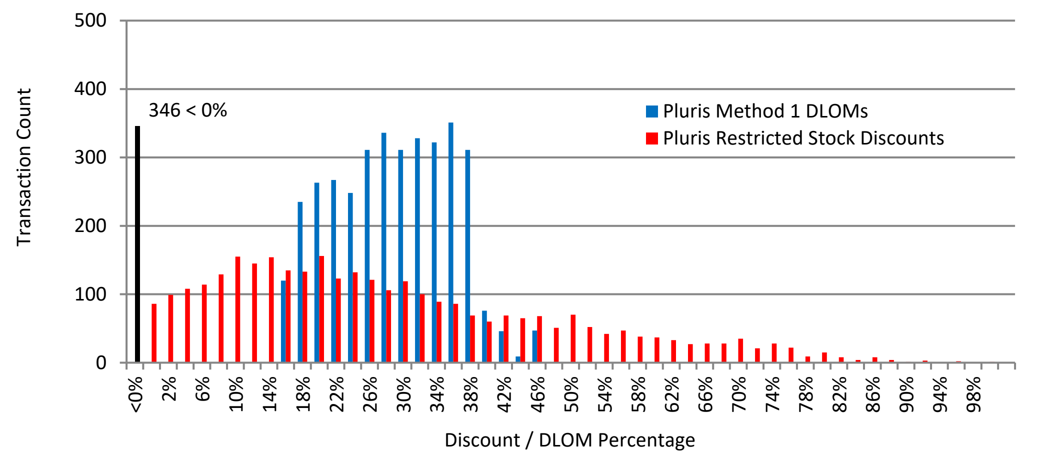
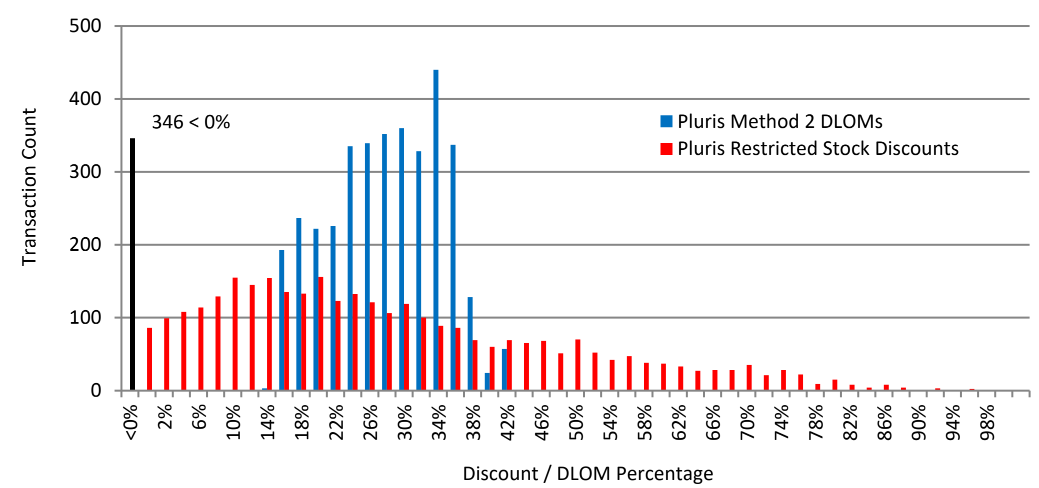


Figure 4.10
Distribution of Discounts and Pluris DLOMs for 3,581 Restricted Stock Transactions
Calculated using Pluris Method 2

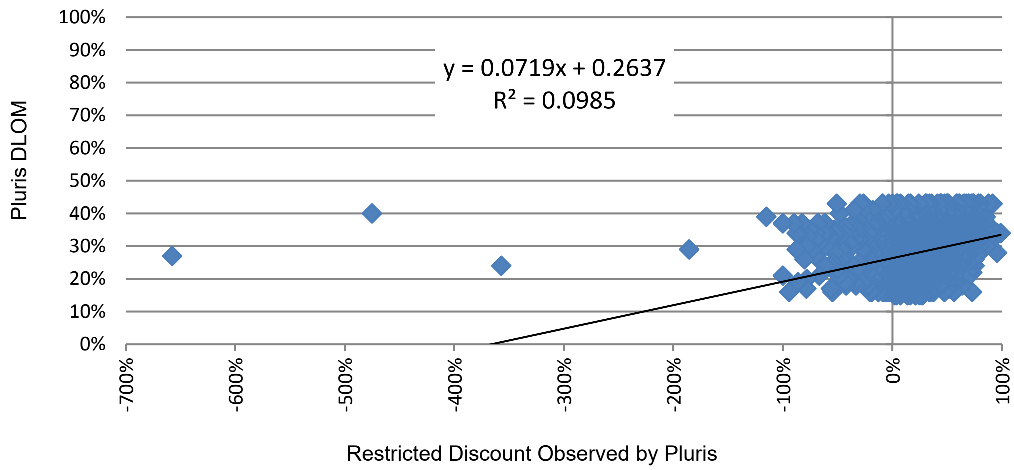


Relying on the Pluris® calculator necessarily means that one is comfortable with the notion that all DLOMs fall within the range of fourteen percent and forty-five percent of the marketable value for all business valuation subjects. One should, instead, be uncomfortable with that notion considering the directly contradictory evidence of negative discounts and the broad range of discounts within the Pluris® DLOM database shown by Figures 4.9 and 4.10.

A matter that should be of great concern to practitioners is that DLOMs calculated using the Pluris® methodologies do not correlate well with the discounts reported for the underlying restricted stock transactions. Figure 4.11 is a scatter graph of the restricted stock discounts reported in the Pluris® DLOM database and the average corresponding DLOMs (RSED + PEDI) of Methods 1 and 2.[70] Figure 4.11 presents the results for 3,581 database transactions for which all of the Pluris® DLOM calculator parameters are present.[71] There is a 9.85% R-square of correlation between the calculated DLOMs and the observed discounts of the underlying transactions. This means that the Pluris® methodology fails to explain over 90% of the variability of the observed discounts.

Figure 4.11

3,581 Pluris® Restricted Stock Premiums and Discounts and Corresponding Pluris® DLOMs



The Pluris® database has 1,743 restricted stock transactions that have complete data for all eight of the specified parameters of the Pluris® calculator and that have no warrants attached. Narrowing the calculations of Pluris® DLOMs to just these transactions resulted in Figure 4.12. It shows a reduced R-square of correlation of 6.45% between the average Method 1 and Method 2 DLOM and the corresponding discount. This leaves almost 94% of the variability of the observed discounts unexplained.

[70] The results are demonstrated using the average values calculated by Pluris Methods 1 and 2 because there is little difference between them. *See* Table 4.18.

[71] Figures 4.11 and 4.12 show four and three transactions, respectively, with extremely negative discounts. These outliers are excluded in Figures 4.13 and 4.14 along with all other negative discounts.

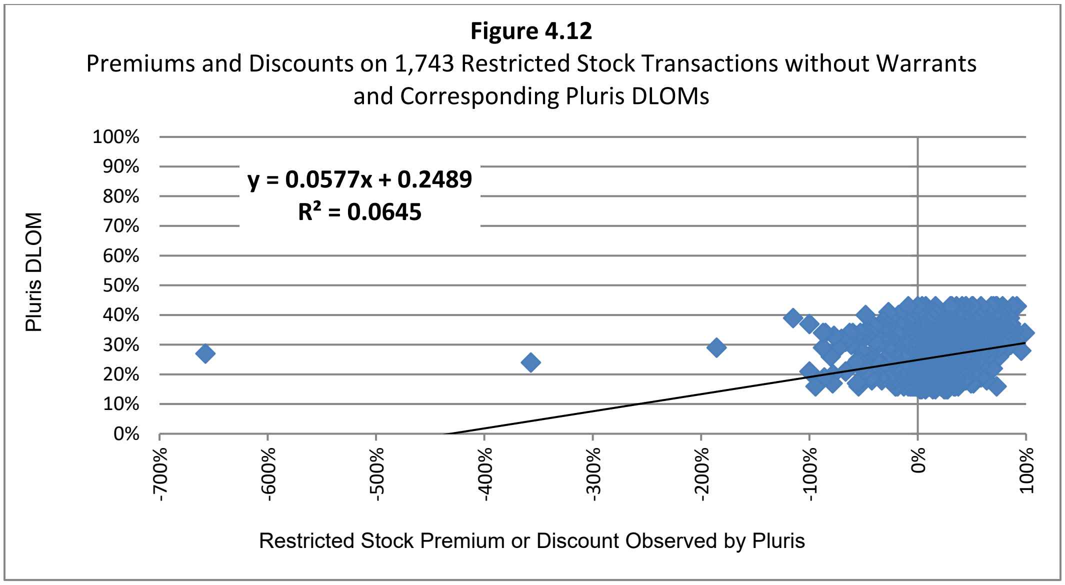


Figure 4.13 presents a scatter graph of a 3,149-transaction subset of the 3,581 shown in Figure 12 that have discounts greater than zero. This dataset includes transactions with and without warrants. The R-square of correlation between the average of the Pluris® Method 1 and Method 2 DLOMs increased to 27.1% for this population of transactions. Still, the Pluris® DLOM methods fail to explain almost 73% of the variability of the discounts of the underlying transactions. Moreover, the x coefficient is an unacceptably low .1746:1; ideally it would be 1:1. And the y intercept is an unacceptably high 23.22%; ideally it would be zero percent.

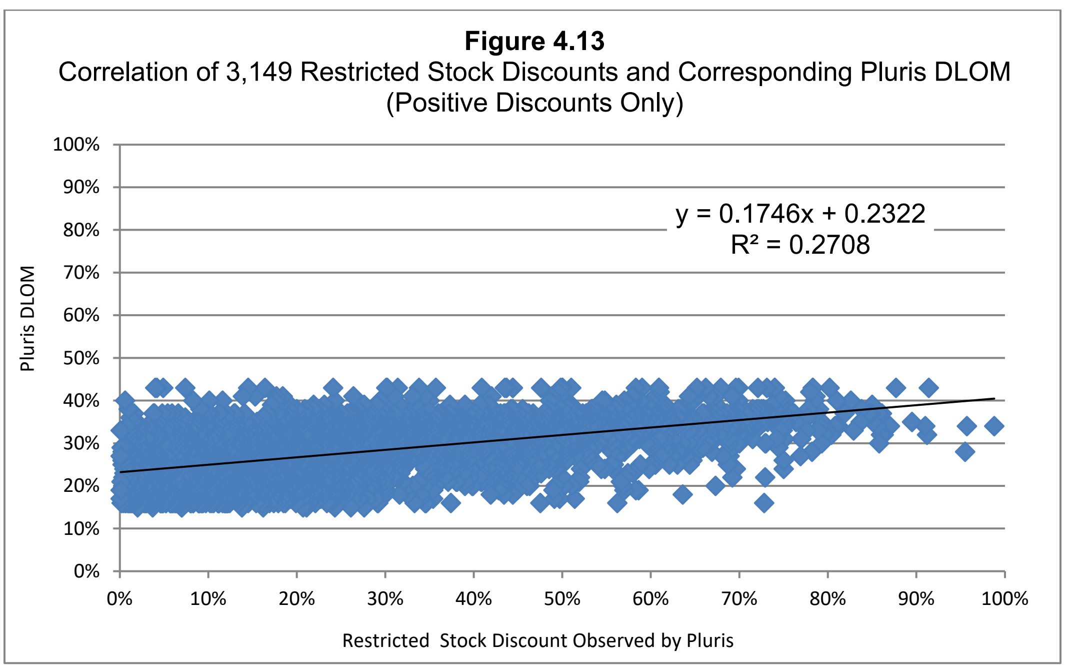
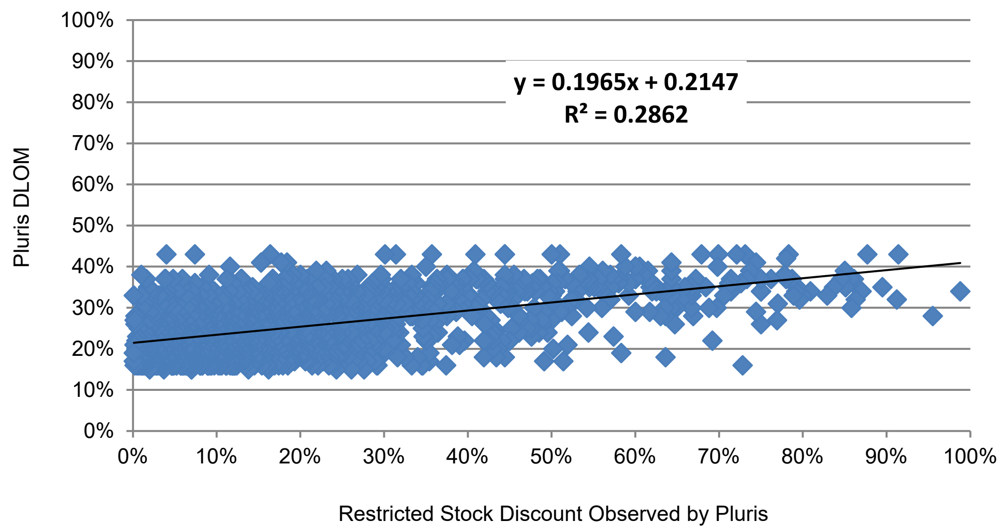


Figure 4.14 presents a scatter graph of the 1,743-transactions shown in Figure 4.11 that do not have warrants and have discounts greater than zero. This sub-population exhibits a slightly more increased R-square of correlation of 28.6%. Nevertheless, the x coefficient remains an unacceptably low .1965:1, and the y intercept remains an unacceptably high 21.47%. The result is that the Pluris® DLOM calculation method squeezes DLOM estimates into a narrow band that is not exhibited by the corresponding discounts.

Figure 4.14
1,743 Warrantless Restricted Stock Discounts and Corresponding Pluris DLOMs
(Positive Discounts Only)



Not surprisingly, Figures 4.11 and 4.12 compared to 4.13, and 4.14 demonstrate a deterioration of correlation between the calculated DLOMs and populations of transactions that include negative discounts. The four Figures also demonstrate the artificial ceiling and floor that the Pluris® methodologies consistently impose on DLOMs regardless of the presence of negative discounts. Figures 4.11 and 4.12 include many transactions with discounts of zero or less, but none of the calculated DLOMs are less than 14%. The similar situation is shown by Figures 4.13 and 4.14 even when the negative discounts are removed; none of the calculated DLOMs approaches 0% value. Despite that a great many of the observed discounts are well above the 45% maximum DLOMs shown by all four Figures.

Of the 3,632 transactions in the full Pluris® database, 1,019 report discounts above the 34.9% maximum RSED and 1,010 report discounts below the 9.4% minimum RSED calculated using the Pluris® DLOM methodologies. *See* Table 4.18. Accordingly, the discounts reported for

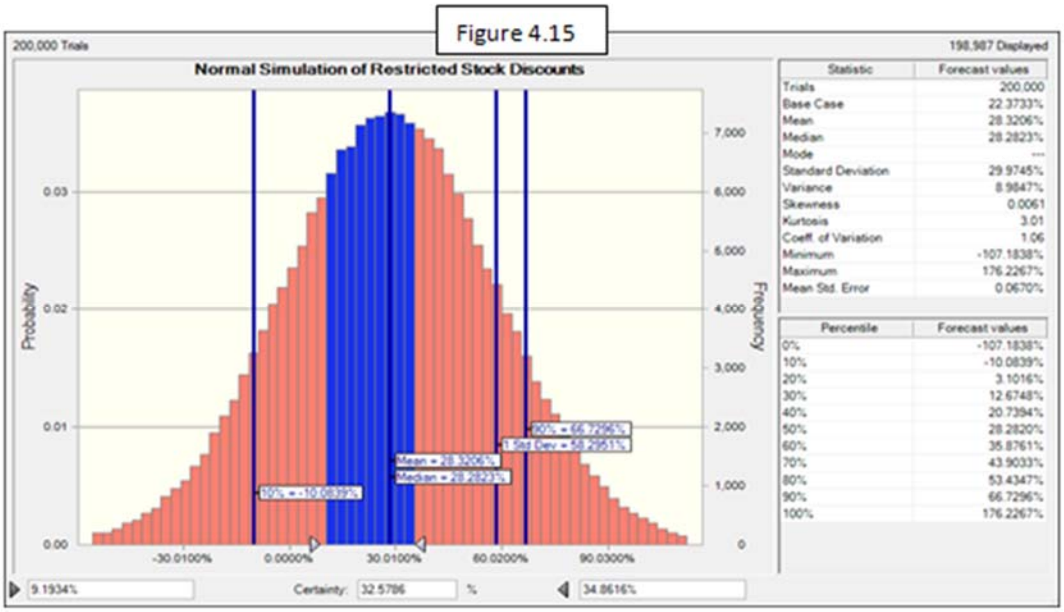
2,029 transactions—fifty-six percent—fall outside the range of the RSED values calculated with the Pluris® methodologies. This fact discloses a serious reliability problem for practitioners.

Oracle Corporation's *Crystal Ball* software was used to evaluate the range of outcomes predicted by the discounts reported in the Pluris® DLOM database. The reported discounts have a statistical mean of 22.37% and a standard deviation of 29.91%.[72] These values were assumed for Monte Carlo simulations. The software performed 200,000-iteration normal (Figure 4.15) and log-normal (Figure 4.16) trials in each simulation.

The normal distribution simulation of Figure 4.15 resulted in a simulated mean of 28.32% and a simulated standard deviation of 29.97%. The simulation shows that the range of outcomes predicted by this mean and standard deviation of the discounts is from *negative* 107% to positive 176% of the publicly traded values. DLOM obviously cannot be less than zero percent nor greater than 100% of the publicly traded price.

The area of Figure 4.15 shown in blue represents the range of RSED values calculated using the Pluris® methods. The Pluris® methods clearly do not predict negative values, nor should they. But Figure 4.15 shows that about 67% of expected discounts would likely fall below the 9.4% minimum and above the 34.9% maximum RSEDs calculated using the Pluris® methods. With a normal distribution assumption there is less than a thirty-three percent certainty that Pluris® RSED methods will yield an accurate value relative to the discounts in the database.

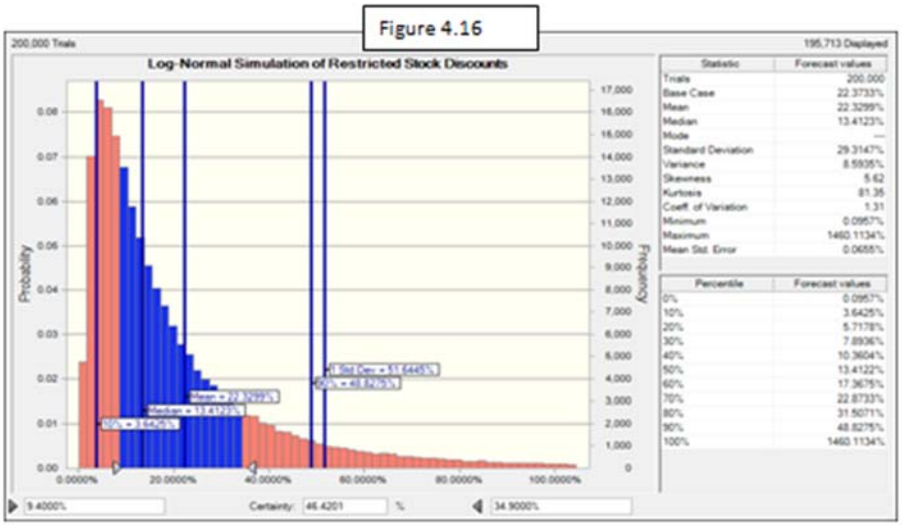
Figure 4.15



[72] Calculated from dlomWebExportData, Pluris® DLOM database Version 4.2.0.

The restricted stock discounts can also be analyzed using a log-normal distribution assumption. The predicted discounts range from zero percent to 1,460%, which is shown with Figure 4.16. Again, the area in blue represents the range of RSED values calculated using the Pluris® methods. The log-normal assumption does not solve the problem of hundreds of actual transactions with negative discounts, but nonetheless, provides useful information by assuming that no forecasted value for DLOM purposes should be less than zero. The log-normal simulation of Figure 15 resulted in a simulated mean of 22.33% and a simulated standard deviation of 29.31% for the discounts of the transaction population. This simulation shows that about seventy percent of the predicted discounts fall below the predicted mean. And this simulation predicts that half of all discounts would fall below and above a 13.4% median of the distribution. The most frequently predicted discount is slightly above 3.6%—well below the minimum RSEDs calculated by Pluris®. On the order of thirty-five percent of discounts are predicted to fall below the 9.4% minimum Pluris® RSED and on the order of fifteen percent to fall above the RSED maximum of 34.9%. About fifty-four percent of expected discounts fall outside the range of RSEDs calculated using the Pluris® methods under the log-normal assumption. *See* Table 4.18 and Figure 4.16. With a log-normal distribution assumption, there is only about a 46% certainty that Pluris® RSED methods yield DLOMs consistent with the restricted stock discounts against which they are benchmarked.

Figure 4.16



Chapter 5
THE STOUT RESTRICTED STOCK STUDY

FMV Opinions®, now known as Stout, released The FMV Restricted Stock Study in 2010. Hereafter, this study is generally referred as The Stout Restricted Stock Study. The names Stout and FMV Opinions® are used interchangeably. This discussion pertains to the 2015 Edition of the Stout (then FMV) Restricted Stock Study 2015. The Study version discussed herein is a database of 769 restricted stock transactions—smaller than the 3,632 transaction Pluris® database discussed in Chapter 4. Similar to the Pluris® product, The Stout Restricted Stock Study offers a DLOM calculator.

The Companion Guide to the 2015 Edition of the Stout (then FMV) Restricted Stock Study states:

> In *Temple v. U.S.*, the court was faced with three different discount approaches: the benchmark average approach, the QMDM (a version of the discounted cash flow approach to determining the DLOM), and the restricted stock comparative analysis approach (RSCAA). The *Temple* court rejected both the benchmark average approach and the QMDM. However, the *Temple* court responded favorably to the RSCAA, stating, "As for the lack of marketability discount, the Court finds [the IRS's expert's] method to be correct…. [T]he Court finds reliability in the fact that [the IRS's expert] endeavored to understand and incorporate the market dynamics of restricted stock sales….The better method is to analyze the data from the restricted stock studies and relate it to the gifted interests in some manner, as [the IRS's expert] did."
>
> Accordingly, the courts have come to a conclusion: the preferred discount methodology is the [Restricted Stock Comparative Analysis Approach]. To use this approach, two things are necessary: (1) a sufficient database of restricted stock transactions, and (2) an in-depth understanding of restricted stock. [[73]]

This chapter considers the reliability of basing DLOM conclusions on benchmarked restricted stock transactions reported in The Stout Study and on DLOMs generated by the Stout DLOM Calculator.

<u>Section 1 — Exploring the Stout Restricted Stock Study</u>

According to Stout, the Stout Restricted Stock Study ("The Stout Study") is a database of private placements of unregistered common stock issued by public companies.[74] Stout relies on a number of sources to identify restricted stock transactions for potential inclusion in its database of transactions. Such sources include: 10K Wizard; Security Data Corp.; EDGAR and EDGAR

[73] *A Companion Guide to the FMV Restricted Stock Study 2015 Edition*, pages 6 and 7.

[74] *Ibid*, page 12.

Pro; Dow Jones News Retrieval; Disclosure CompactD; and S&P Corporate Transactions Records.[75] The version of The Stout Study considered here comprises 769 individual restricted stock transactions. In addition to the stock issuer's name and stock ticker symbol, there are potentially 52 fields of data associated with each restricted stock transaction. Data is reported in the majority of fields for the majority of listed transactions.

Stout states that it conducts a thorough review of all relevant public filings and filing exhibits associated with a restricted stock transaction, reviewing thousands of transactions that are winnowed down to those included in The Stout Study.[76] The transaction selection protocol stated by Stout appears to be robust, resulting in the elimination of 95% of all transactions reviewed.[77] The following types of transactions are excluded from The Stout Study[78]:

1. Stock placements that were registered prior to the transaction date or that became registered within 30 days of the transaction date;
2. Placements of stocks that are not identical to common stock, such as hybrid securities that include debt, preferred stock, convertible preferred stock, or some kind of hybrid equity-derivative;
3. Placements of stocks that include warrants;
4. Stock placement transactions that did not close;
5. Placements of stocks that are traded exclusively on non-U.S. stock exchanges;
6. Placements of stocks for which the registered equivalent traded below $1 per share for the entire month of the transaction, or that had extremely low trading volume;
7. Transactions for which significant pieces of information are unavailable (e.g., the market reference price, the private placement transaction price, or the gross purchase price per share);
8. Stock placements that included special contractual arrangements between the buyer and the seller;
9. Stock placements that occurred as part of another transaction; and
10. Stock placements more than 50% of which were to parties related to the issuer.

No attempt to test the accuracy of the data presented in The Stout Study; accuracy was assumed for the purposes of the work herein. Practitioners should consider independently verifying the data reported for specific transactions on which they intend to rely.

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[75] *Ibid*

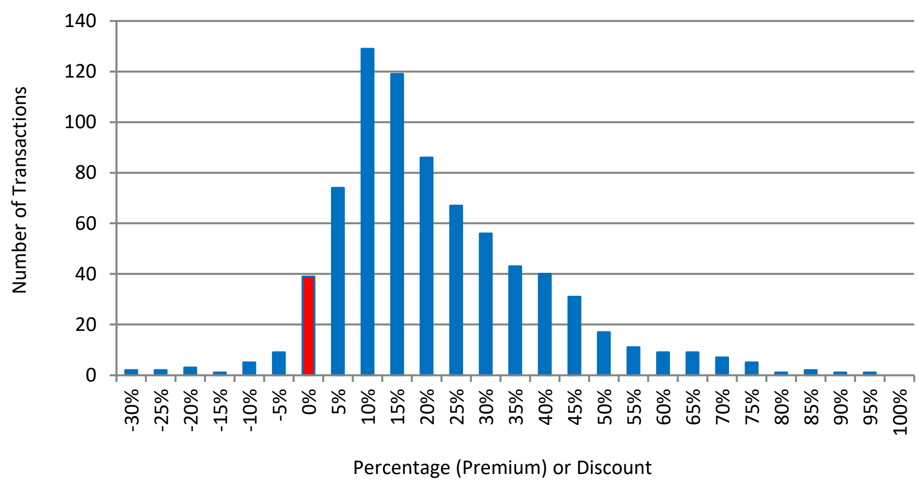
[76] *Ibid*

[77] *Ibid*

[78] *Ibid*

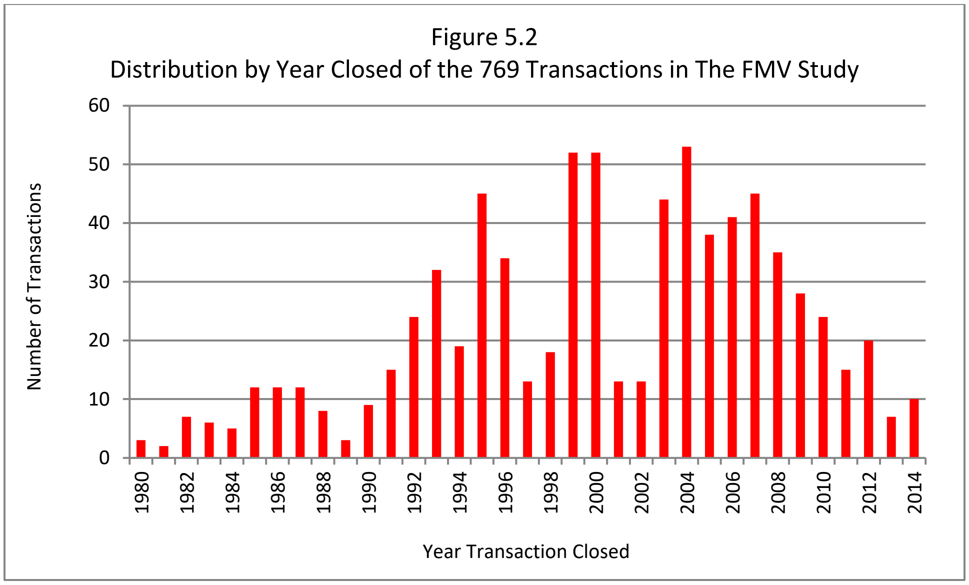
Of the 769 transactions comprising The Stout Study, not all reflect discounts: 42 restricted stocks were sold at price premiums; 19 were sold at prices equal to the publicly traded price; and 708 were sold at price discounts. Figure 5.1 shows the distribution of restricted stock price premium and discount reported in The Stout Study.

Figure 5.1
Distributions of Premiums and Discounts of the 769 Transactions in The FMV Study



Chapter 4 explained the unlikely expectation that a valuation subject could reasonably be benchmarked against one of the 3,632 transaction reported in the Pluris® database. Identifying a reasonable benchmark among the 769 transactions in the Stout database is even more unlikely. These 769 transactions represent just 595 unique stock issuers, of which 125 floated 299 of the reported transactions. Thus, the negotiations of 21% of stock issuers determined 39% of the discount conclusions represented by the Stout database.

The oldest transaction in The Stout Study closed July 1, 1980, and the most recent closed August 13, 2014—a span of about 35 years. Figure 5.2 shows the annual distribution by closing date of the 769 transactions listed in the database. The average number of transactions annually in The Stout Study is about 22, or fewer than 2 per month. But the annual range in number of transactions is significant. For example, fewer than 10 transactions occurred annually in years 1980 through 1984, 1988 through 1990, and 2013, while more than 50 transactions occurred annually in years 1999, 2000, and 2004.



Matching a subject company valuation temporally to even one transaction in The Stout Study can be a daunting task. The numerically limited transactions necessarily mean that no transaction is reported for the vast number of dates in the 35 year span of The Stout Study—a period of 12,775 days. The chance of temporally matching a valuation date to a transaction closing date would be 6% if each transaction in the study occurred on a unique date. But 257 transaction dates are not unique, and only 617 individual closing dates are represented in The Stout Study, as Table 5.1 shows. Reducing the number of days to the approximate number of stock-trading days during the database period—about 8,750 days—makes little improvement in the probability of matching a valuation date to a restricted stock transaction date.

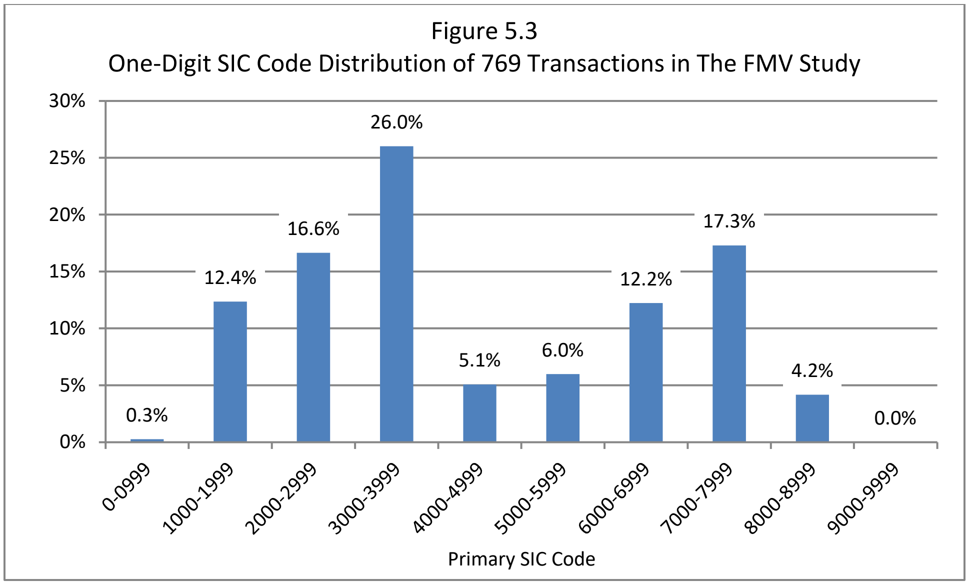
The date matching problems increase dramatically if one desires more than one transaction for a particular date as Table 5.1 also shows. The fact that most valuation dates are relatively recent while most of the transaction dates are many years old exacerbates the temporal problems of The Stout Study.

Table 5.1

Number of Stout Study Restricted Stock Transactions Closing on a Date in the Stout Database

	All Database Days		Stock Trading Days	
Number of Occurrences	Number of Days	Percent of Days	Number of Days	Percent of Days
0	12,158	95.17%	8,133	92.96%
1	512	4.00%	512	5.85%
2	75	0.59%	75	0.86%
3	17	0.13%	17	0.19%
4	10	0.08%	10	0.11%
5	2	0.02%	2	0.02%
6	1	0.01%	1	0.01%
Total Days	12,775	100.00%	8,750	100.00%

The number of industries represented in The Stout Study is also very limited. Figure 5.3 shows the distribution of the 769 transactions in database by primary SIC code. It is readily observed that The Stout Study is highly concentrated in the 3000 SIC code series, accounting for 200 transactions. The 3000 series of SIC codes primarily represents manufacturing products made from rubber, plastics, leather, stone, clay, glass, concrete, and primary metal; and fabricated products such as industrial and commercial machinery, computer equipment, electronic equipment and components, transportation equipment, and technical equipment products. Two other SIC code series account for substantial numbers of transactions in The Stout Study—the 7000 and 2000 series. The Stout Study has 133 transactions in the 7000 series, which represents hotels and other lodging places; personal, business, automotive-related, and miscellaneous repair services; motion pictures; and amusement and recreation services. The Stout Study has 128 transactions in the 2000 series, which represents manufacturing of products such as food, tobacco, textiles, apparels, lumber and wood, furniture and fixtures, paper and paper products, printing and publishing, chemicals, and petroleum refining. Together these three series account for virtually 60% of the transactions comprising The Stout Study.



But the transactions comprising The Stout Study are actually even more concentrated than described in the previous paragraph. Over 70% of the transactions fall into eight two-digit SIC codes, as summarized in the Table 5.2:

Table 5.2
Number of Transactions in The Stout Study by Two-Digit SIC Code

SIC Code	Industry Description	Number	Population Percentage
2800-2899	Manufacturers of Chemicals and allied products	111	14.4%
7300-7399	Business services	107	13.9%
3800-3899	Manufacturers of measuring, analyzing, and controlling instruments; Photographic, medical, and optical goods; Watches and clocks	84	10.9%
1300-1399	Oil and gas extraction	75	9.8%
3600-3699	Manufacturers of electronic and other electrical equipment and components, except computer equipment	71	9.2%
6000-6099	Depository institutions	45	5.9%
3500-3599	Manufacturers of industrial and commercial machinery and computer equipment	28	3.6%
6700-6799	Holding and other investment offices	28	3.6%
	Total transactions in the eight SIC codes above	549	71.3%
	All other 83 two-digit SIC codes in The Stout Study	220	28.7%
	All transactions in The Stout Study	769	100.0%

There are an estimated 1,008 unique four-digit SIC codes of which 176 (17%) are represented in The Stout Study. Accordingly, no transactions are reported for 83% of SIC codes. Furthermore, more than half of all transactions included in The Stout Study fall into just 18 four-digit codes, which is less than 2% of all SIC industries. Table 5.3 presents the 18 SIC codes.

Table 5.3
Number of Transactions in The Stout Study by Four-Digit SIC Code

SIC Code	Industry Description	Number
1311	Crude Petroleum & Natural Gas	59
2834	Pharmaceutical Preparations	59
7372	Services-Prepackaged Software	46
2836	Biological Products, (No Diagnostic Substances)	24
6022	State Commercial Banks	24
3841	Surgical & Medical Instruments & Apparatus	22
7373	Services-Computer Integrated Systems Design	17
3674	Semiconductors & Related Devices	16
3845	Electromedical & Electrotherapeutic Apparatus	16
6021	National Commercial Banks	16
2835	In Vitro & In Vivo Diagnostic Substances	15
3663	Radio & TV Broadcasting & Communications Equipment	14
3842	Orthopedic, Prosthetic & Surgical Appliances & Supplies	11
6712	Offices of Bank Holding Companies	11
7371	Computer Programming Services	11
1041	Gold Ores	10
3679	Electronic Components, not elsewhere classified	10
7812	Motion Picture and Video Tape Production	10
Total transactions in the 18 SIC codes above		391
Total transactions in the other 158 four-digit SIC codes in The Stout Study[79]		378
All transactions in The Stout Study		769

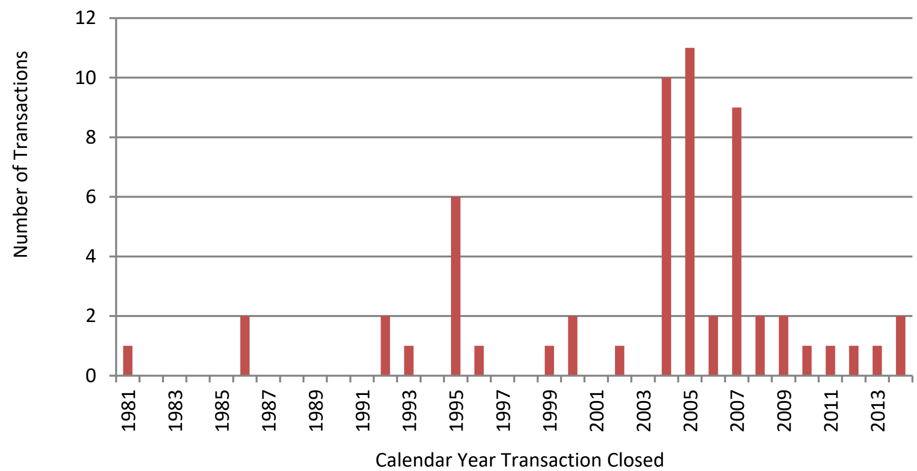
It should be obvious to readers that it would be extremely difficult to find a transaction in The Stout Study that is in a subject company's industry much less a sufficiently comparable transaction against which to directly benchmark a DLOM both temporally and characteristically. For example, the 59 transactions representing SIC code 1311 have transaction closing dates ranging from March 1, 1981, to June 18, 2014—a period of 12,162 days (8,687 week days). The

[79] Only 24 of these industry codes have at least five transactions, which is the minimum necessary to satisfy a quintile methodology such as Pluris® uses in its DLOM calculator.

chance of finding an SIC code 1311 transaction that occurred on a specific week day valuation date is 1 in 147 (i.e., 0.7%), assuming that each of the 59 code 1311 transactions in The Stout Study occurred on a different day.

Concentration of the timing of transactions adds to the difficulty. For example, referring to Figure 5.4, 11 transactions closed in calendar year 2005 and 10 in 2004. These years represent 36% of the SIC code 1311 transactions and no transactions occurred in calendar years 1982-1985, 1987-1991, 1994, 1997-1998, 2001, and 2003.

Figure 5.4
Distribution of Closing Dates of SIC Code 1311 Transactions in The FMV Study



Concentration of issuers further diminishes the benchmarking utility of The Stout Study. There are 45 issuers of the 59 transactions comprising SIC code 1311 in the database. Thirty-seven of those issuers were unique issuers of restricted stock. The other 8 non-unique issuers (about 18% of the 45 issuers with the 1311 code) closed 22 restricted stock transactions, approaching forty percent of the 59 transactions. Moreover, these 22 transactions are themselves concentrated: Harken Energy Corporation was the issuer of 5 transactions; BMB Munai, Inc. was the issuer of 4 transactions; and MarkWest Energy Partners, L.P. was the issuer of 3 transactions.

The chance of finding at least one transaction in a single month for a particular four-digit SIC code is roughly 1 in 423,360.[80] The chance is about 1 in 12.7 million of finding an industry-matching transaction on a particular date such as a valuation date. Broadening the definition of comparability to include 12 months improves the chances of finding a theoretically comparable

[80] 35 years x 12 months x 1,008 SIC codes = 423,360 combinations.

transaction to about 1 in 35,000—still very poor odds. Meanwhile, expanding the allowable range of SIC codes requires deciding that different industries are comparable to the subject industry.

The lack of comparability of reported restricted stock transactions to a valuation subject was discussed in Chapter 4 regarding the Pluris® database. As with the Pluris® database, the difficulty of finding sufficiently comparable transaction is potentially fatally undermined by the fact that nothing is known about how the transaction discount (or premium) was actually determined. This deficiency risks appraisers failing to consider the same things considered by the stock negotiators. This point is driven home by the fact that The Stout Study reports several instances of transactions of the same issuer occurring on the same day but with different percentage discounts. *See* Table 5.4. The considerations that led to these different discounts are unknown.[81]

Table 5.4
Same Day Transactions With Different Discounts in the Stout Database

Transaction ID Number	Issuing Company	Closing Date	Transaction Discount Reported by Stout
72	Candie's, Inc.	5/1/1994	27.18%
71	Candie's, Inc.	5/1/1994	34.47%
97	Chief Consolidated Mining Company	4/2/1999	49.57%
96	Chief Consolidated Mining Company	4/2/1999	27.95%
64	Brilliant Digital Entertainment, Inc.	5/1/1999	62.11%
63	Brilliant Digital Entertainment, Inc.	5/1/1999	65.40%
49	Authentidate Holding Corp.	2/4/2004	12.05%
48	Authentidate Holding Corp.	2/4/2004	12.36%
693	Procera Networks, Inc.	9/12/2008	14.39%
692	Procera Networks, Inc.	9/12/2008	20.86%

Unlike the public marketplace for which it can be presumed that investors' decisions are based on publicly available information, restricted stock pricing is a matter of private negotiation. It is therefore speculation that the motivations of the actual private negotiators aligned with any particular piece of information selected by an appraiser for benchmarking. It is further problematic that narrowing transaction selection criteria in the pursuit of comparability inhibits the

[81] The Stout Study also includes two restricted stock issuers (Perficient, Inc. and SmartServ Online, Inc.) for whom two transactions occurred on the same day. In those instances the reported transactions discounts of the issuers were the same.

ability to find any potentially relevant transaction to use as a benchmark for a valuation subject, while expanding the selection criteria to find more transactions introduces potentially countless unknown variables.

Section 2 — The Association of Certain Company Statistics and Restricted Stock Discounts

The following statement and table are extracted from the *Companion Guide to The FMV Restricted Stock Study*:[82]

> [L]ower market values, revenues, total assets and book values, and higher market-to-book (MTB) ratios and stock price volatility are correlated with higher discounts. Accordingly, higher investment risk, as reflected in smaller firm size, higher MTB ratios, and increasing stock price volatility, tends to increase the discount. Profitability is also often used as an indicator of firm risk. However, absolute levels of earnings/losses do not demonstrate a strong correlation with the discount due primarily to the greater impact of company size on the discount. Private placements by large, unprofitable firms tend to exhibit lower discounts than small, profitable firms. Net profit margin tends to be a better indicator that net income as it is not impacted by firm size.[[83]]

Exhibit 5. Comparison of Company Characteristics Between High-Discount Transactions and Low-Discount Transactions[84]

Quintile[1]	1	2	3	4	5
Discount					
Low	0.0%	7.5%	13.1%	20.9%	33.9%
High	7.4%	13.0%	20.8%	33.5%	91.3%
Median	4.1%	10.0%	16.2%	26.2%	43.2%
Company Characteristics (Median Statistics)[2]					
Market Value ($mm)	178.6	192.6	113.7	101.4	56.7
Revenues ($mm)	31.1	41.2	22.8	17.0	8.3
Total Assets ($mm)	112.1	83.2	37.2	23.0	11.2
Book Value of Equity ($mm)	49.3	41.1	20.2	13.6	6.4
MTB Ratio	2.8	3.6	3.6	5.7	6.2
Net Income ($mm)	(4.5)	(1.9)	(3.0)	(4.4)	(2.6)
Net Profit Margin	-6.7%	-5.6%	-6.6%	-22.3%	-39.1%
Volatility	64.1%	65.4%	73.0%	80.2%	104.0%
VIX	18.0	17.6	17.5	18.0	21.3

1) Transactions sorted by discount. Each "quintile" includes 145 or 146

[82] *A Companion Guide to the FMV Restricted Stock Study 2015 Edition*, page 14.

[83] Id.

[84] *Ibid,* page 14, Exhibit 5.

transactions.
2) All statistics have been adjusted for inflation as of January 2015.
3) Premiums have been excluded from this analysis.

FMV Opinions did not present any regression results to support its correlation statements. That analysis is therefore presented below with graphs that generally exhibit high R-squares of correlation for the quintile groupings shown in the FMV table. The exception is net income, which has a very low R-square of correlation. Regardless, the predictions of the regression formulas are generally illogical for DLOM purposes. Consider:

- The R-square of correlation of the quintile groupings of median discounts and market value for is 93.4%, as shown in Figures 5.5A and 5.5B. Consistent with general expectations the regression line of Figure 5.5B is negative, meaning that discounts decline as market value increases. But the regression formula results in 0% discounts for businesses with market values of $227.5 million or more and a 100% discount for those with market values of $830,000 or less.

Figure 5.5A
Comparing Quintile Groupings of Market Value and Median Discounts of The FMV Study

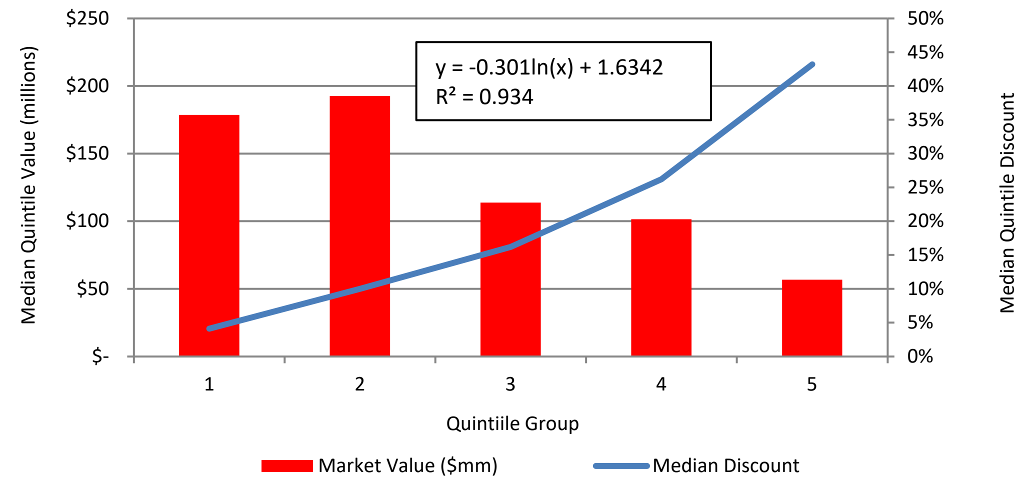
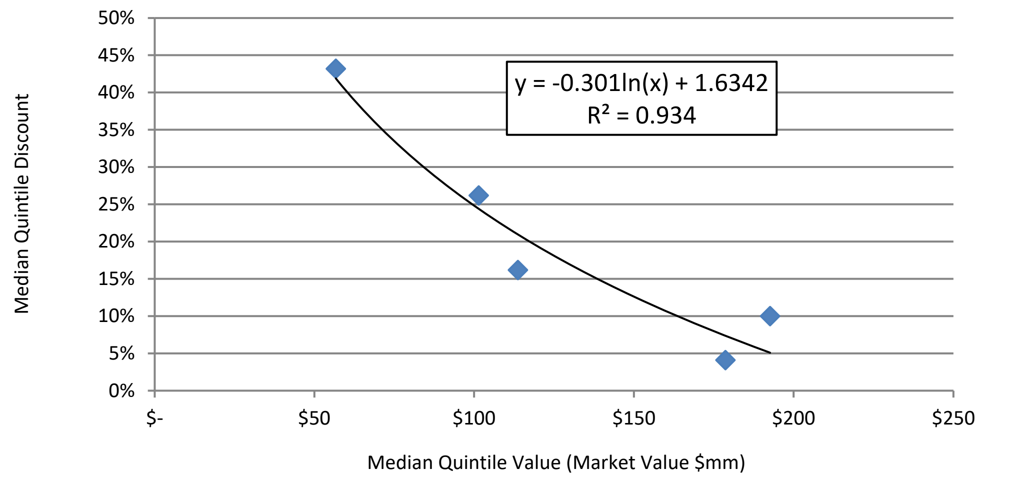
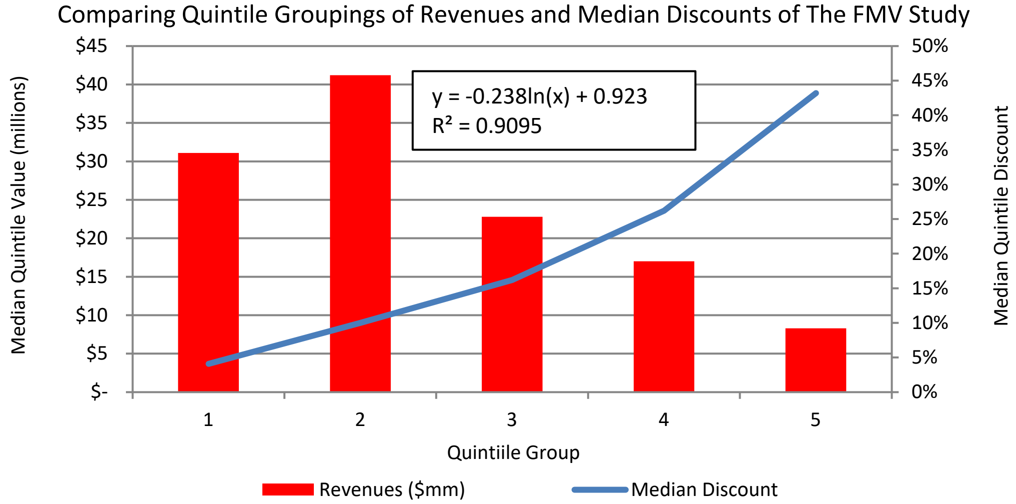


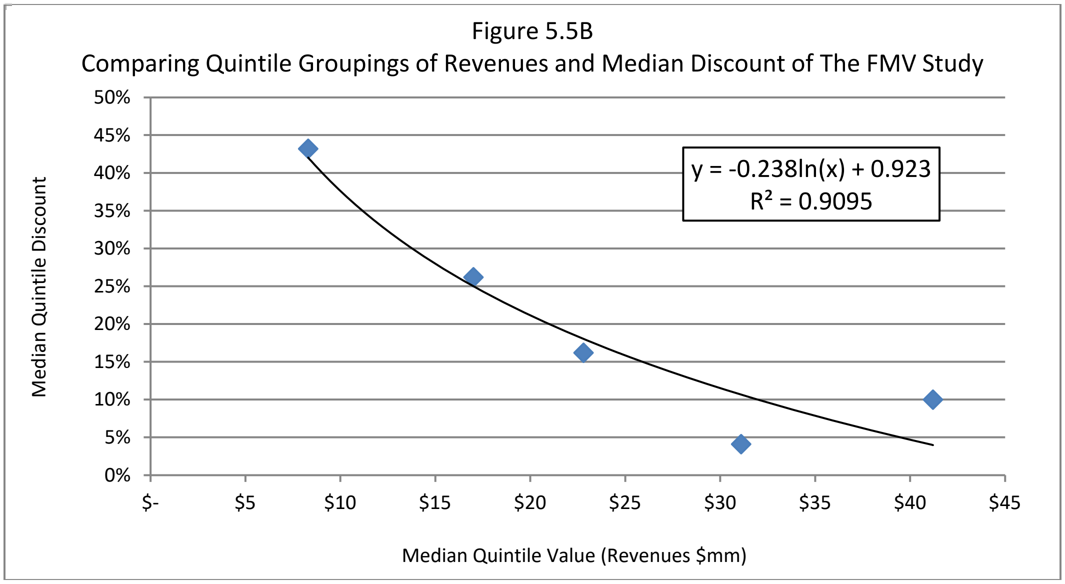
Figure 5.5B
Comparing Quintile Groupings of Market Value and Median Discounts of The FMV Study



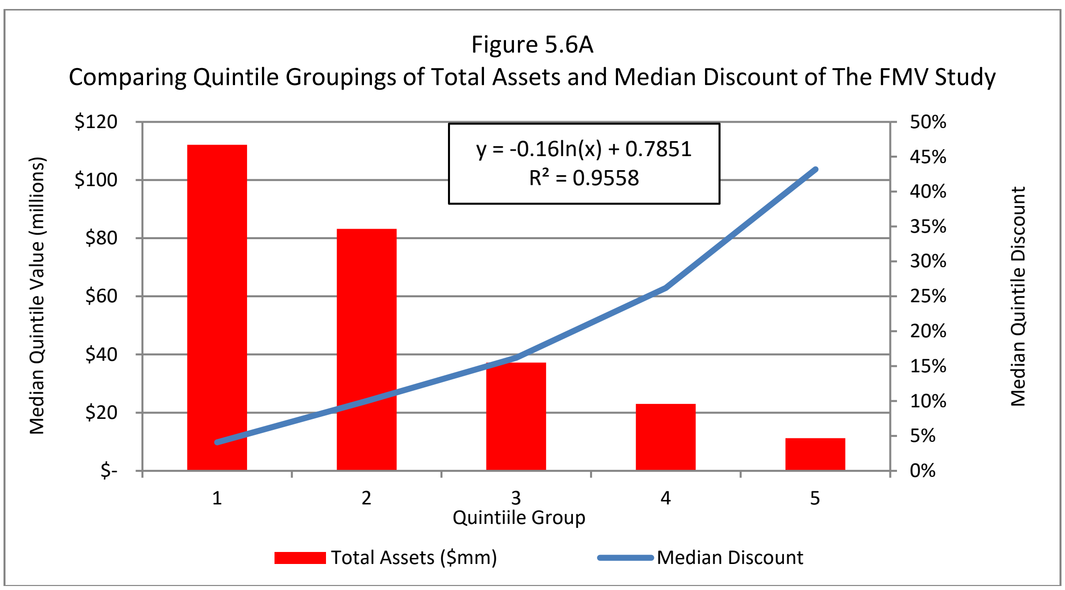
- The R-square of correlation of the quintile groupings of median discounts and revenues is 91.0%, as shown in Figures 5.6A and 5.6B. Consistent with general expectations the regression line of Figure 5-6B is negative, meaning that discounts decline as revenues increase. But the regression formula results in 0% discounts for businesses with revenues of $48.5 million or more and a 100% discount for those with revenues of $710,000 or less.

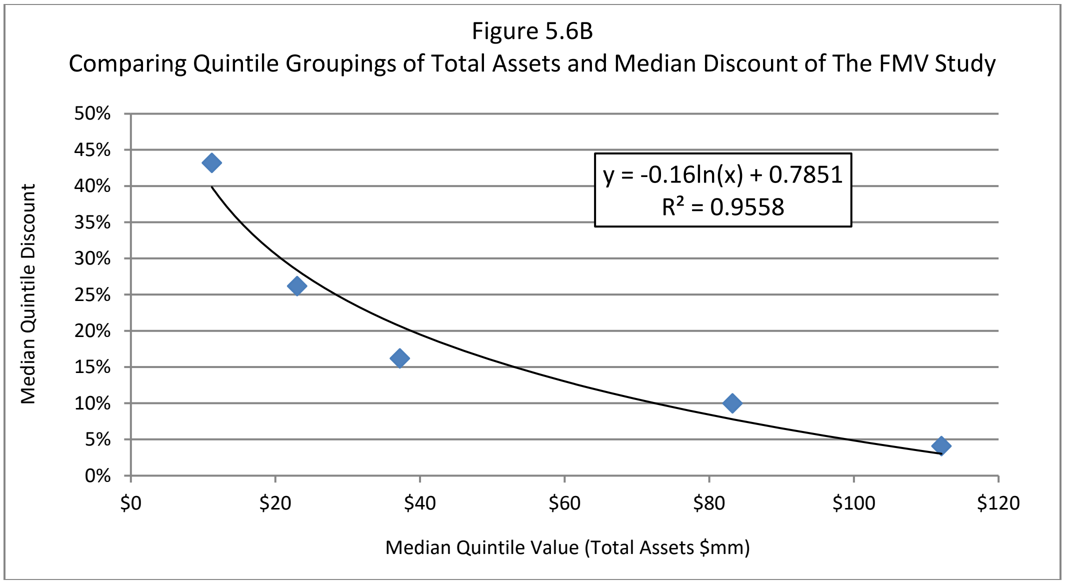
Figure 5.5A
Comparing Quintile Groupings of Revenues and Median Discounts of The FMV Study





- The R-square of correlation of the quintile groupings of median discounts and total assets is 95.6%, as shown in Figures 5.6A and 5.6B. Consistent with general expectations the regression line of Figure 5.6B is negative, meaning that discounts decline as total assets increase. But the regression formula results in 0% discounts for businesses with total assets of $136 million or more and a 100% discount for those with total assets of $260,000 or less.





- The R-square of correlation of the quintile groupings of median discounts and net book value is 96.9%, as shown in Figures 5.7A and 5.7B. Consistent with general expectations the regression line of Figure 5.7B is negative, meaning that discounts decline as total assets increase. But the regression formula results in 0% discounts for businesses with net book values of $62 million or more and a 100% discount for those with total assets of $250,000 or less.

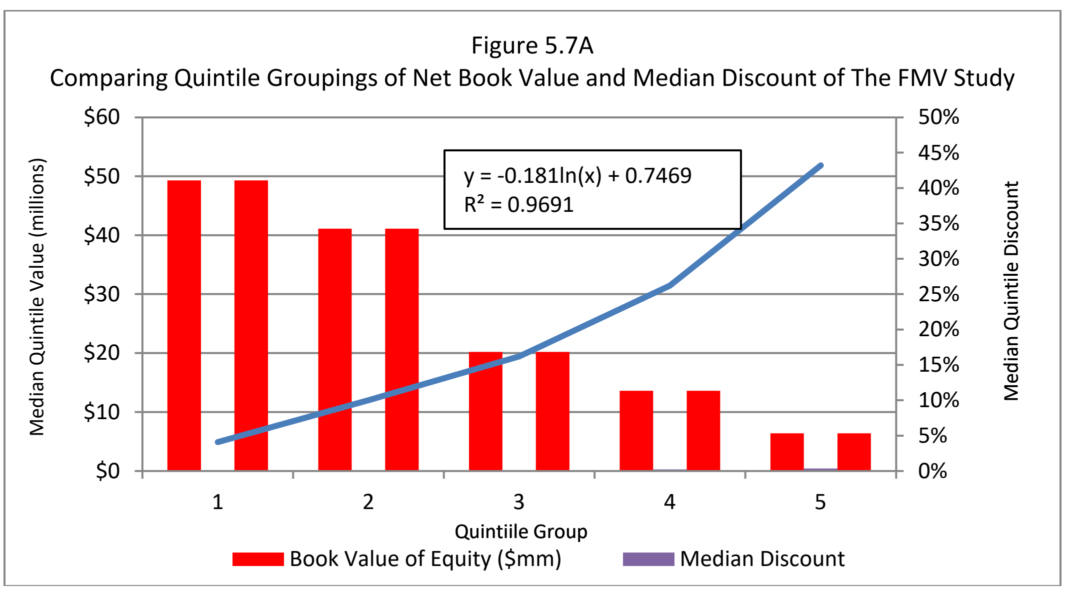
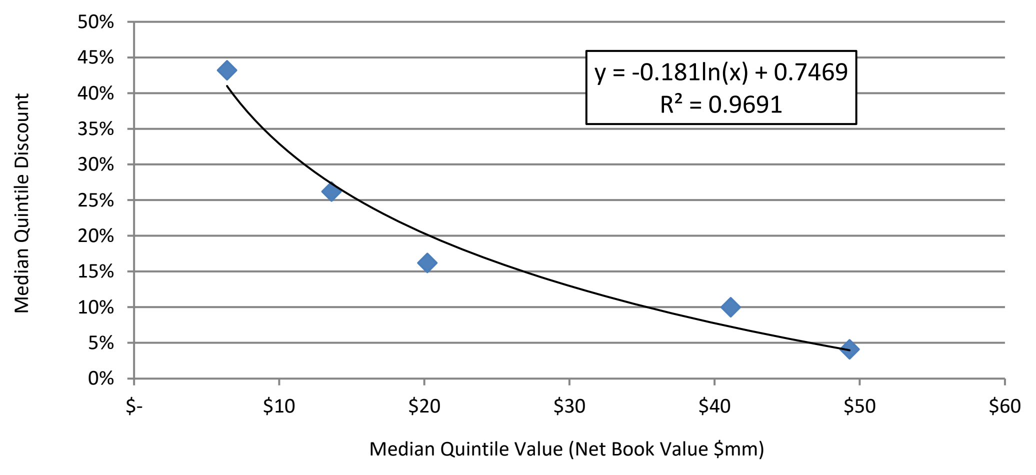


Figure 5.7B
Comparing Quintile Groupings of Net Book Value and Median Discount of The FMV Study



- The R-square of correlation of the quintile groupings of median discounts and market-to-book ratios is 88.2%, as shown in Figures 5.8A and 5.8B. Consistent with general expectations the regression line of Figure 5.8B is positive, meaning that discounts increase as market-to-book ratios increase. But the regression formula results in 0% discounts for businesses with market-to-book ratios of 2.625x or less and a 100% discount for those with market-to-book rations of 27x or more.

Figure 5.8A
Comparing Quintile Groupings of Market-to-Book Ratio and Median Discount of The FMV Study

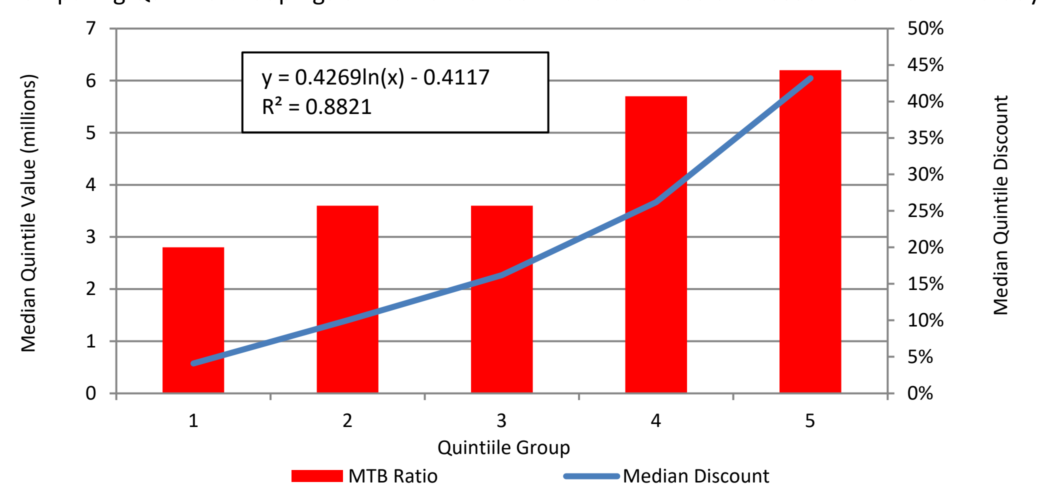
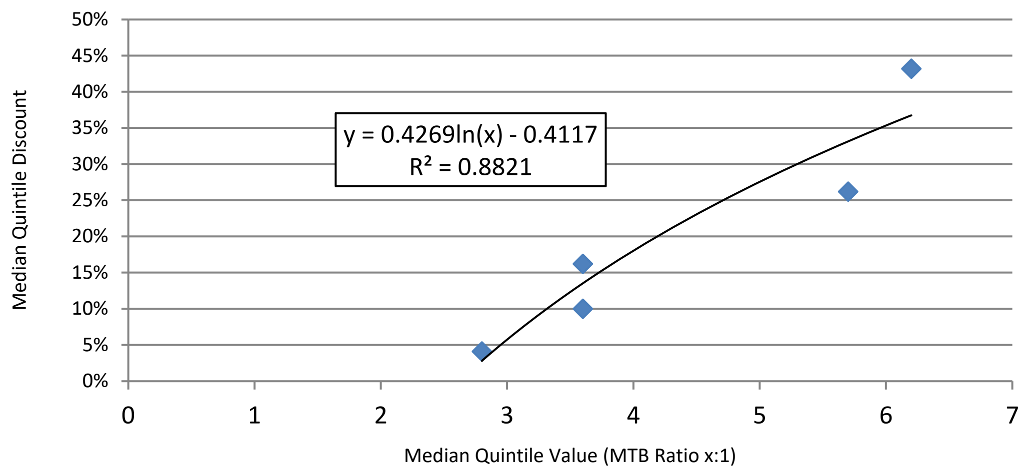
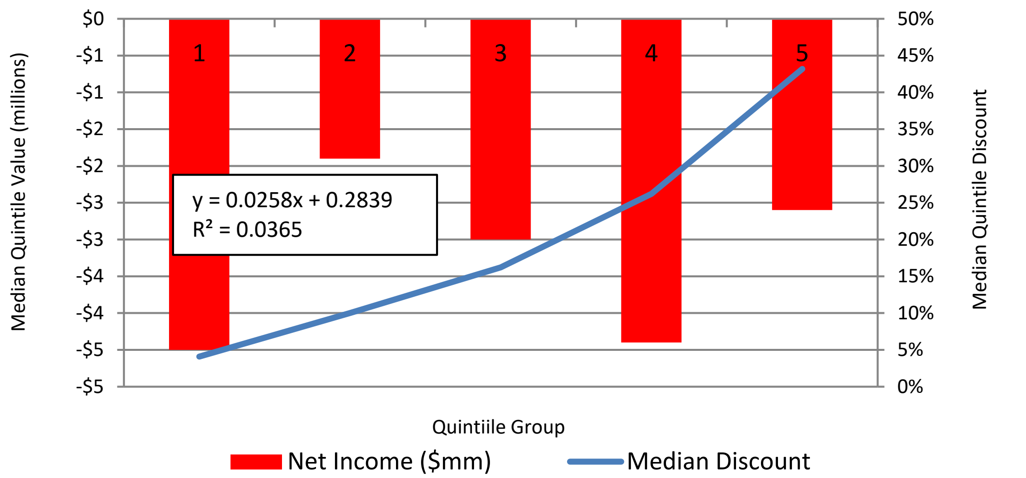


Figure 5.8B
Comparing Quintile Groupings of Market-to-Book Ratio and Median Discount of The FMV Study

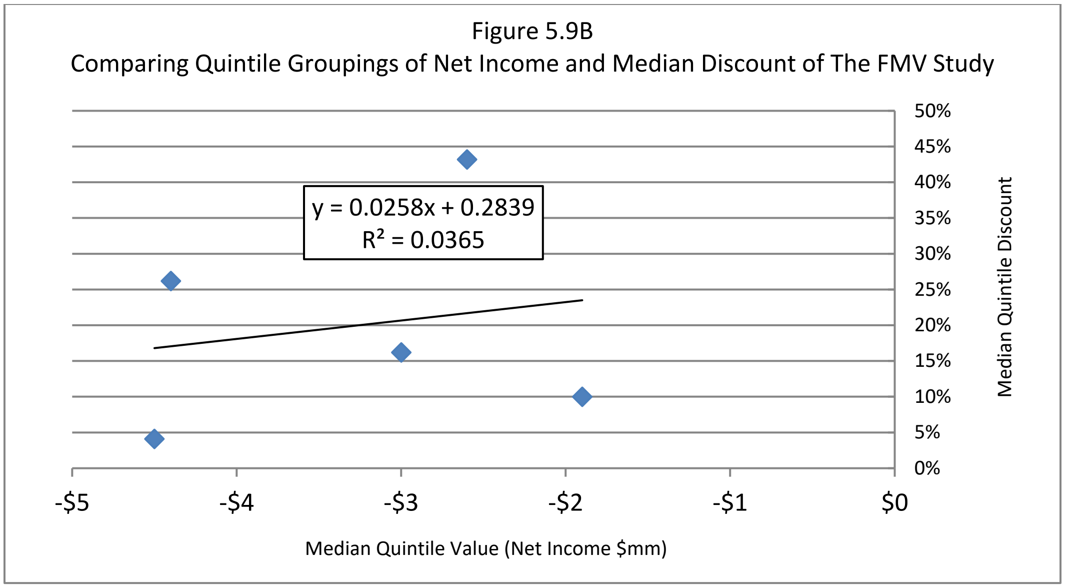


- The R-square of correlation of the quintile groupings of median discounts and net income is a very low 3.65%, as shown in Figures 5.9A and 5.9B. This association illogically results in higher discounts as net income increases.[85]

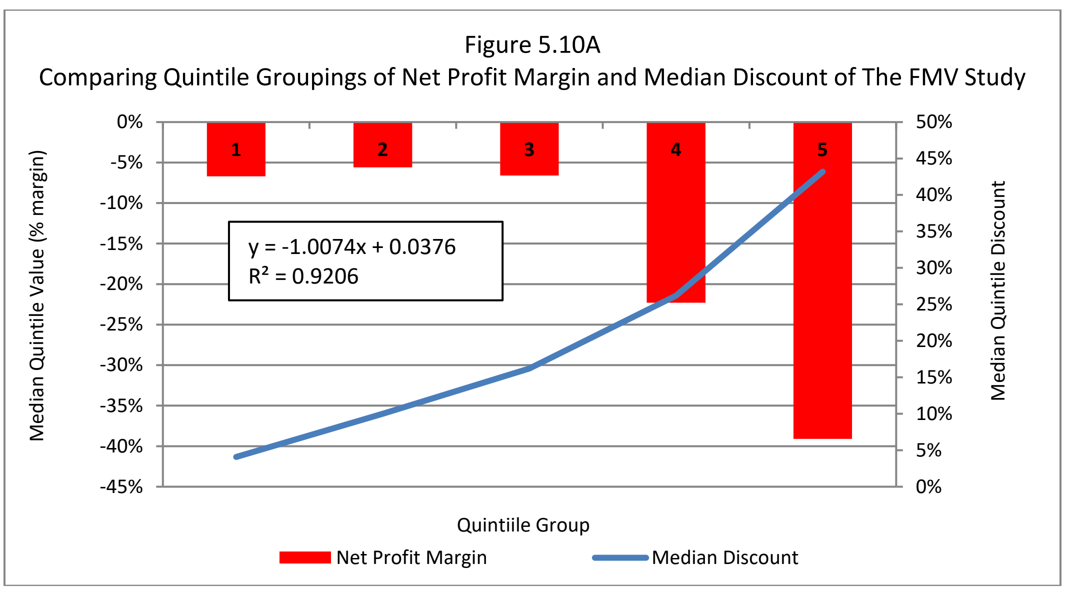
Figure 5.9A
Comparing Quintile Groupings of Net Income and Median Discount of The FMV Study

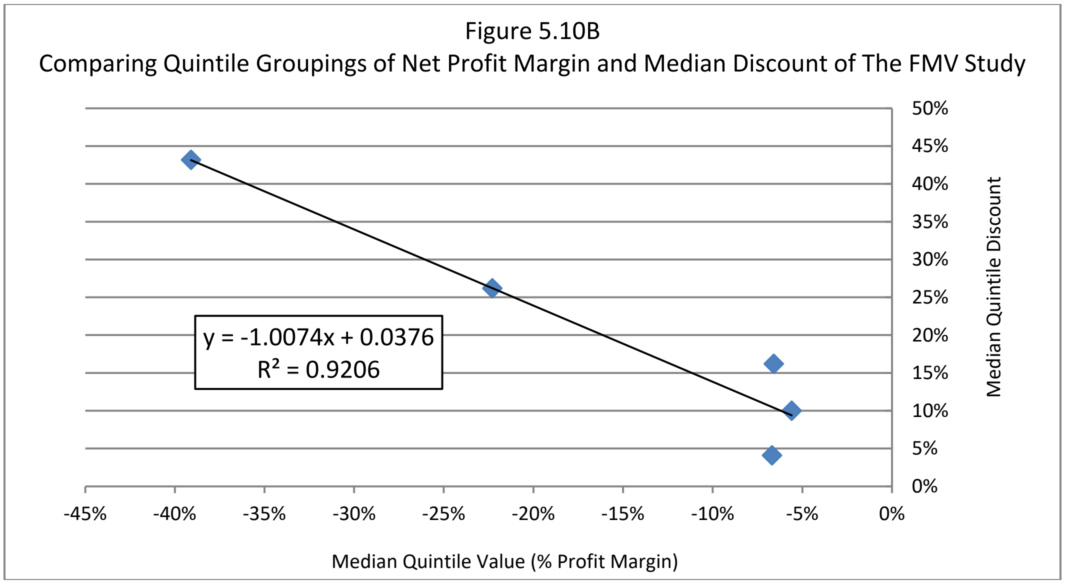


[85] The regression formula is linear because negative values cannot be regressed logarithmically.

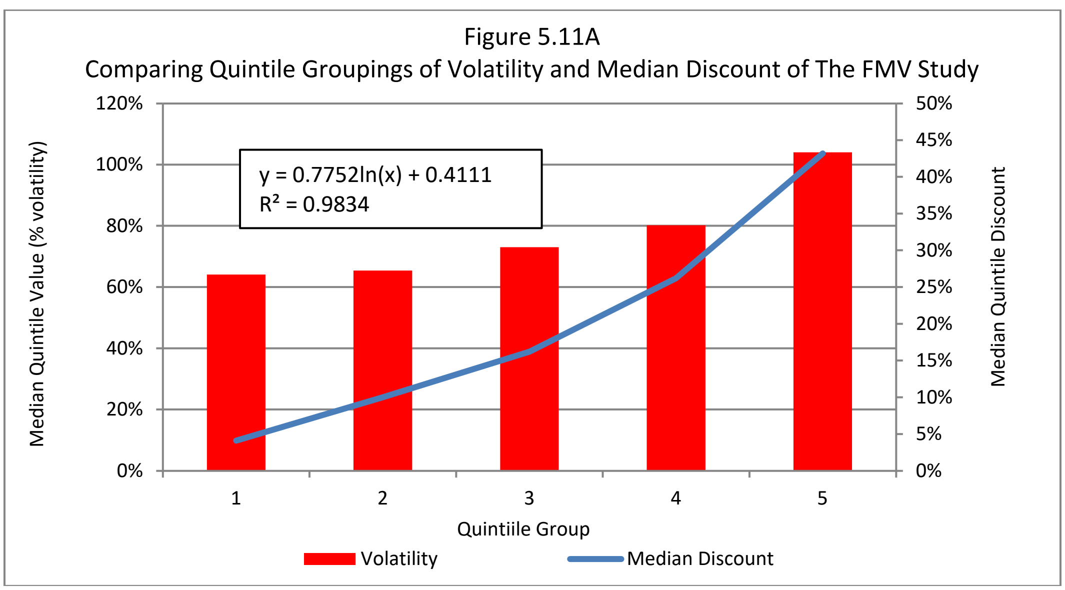


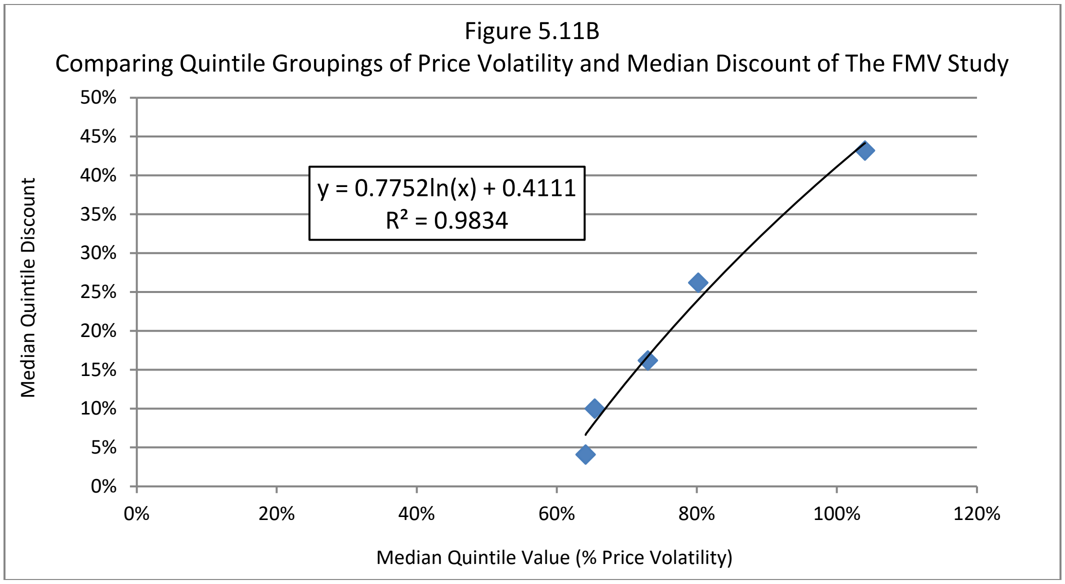
- The R-square of correlation the quintile groupings of median discounts and net profit margins is 92.1%, as shown in Figures 5.10A and 5.10B. Consistent with general expectations the regression line of Figure 5.10B is negative, meaning that discounts decline as net profit margins increase. But the regression formula results in 0% discounts for businesses with net profit margins of 3.725% or more and 100% discounts for those with net profit margins of -96% or less.



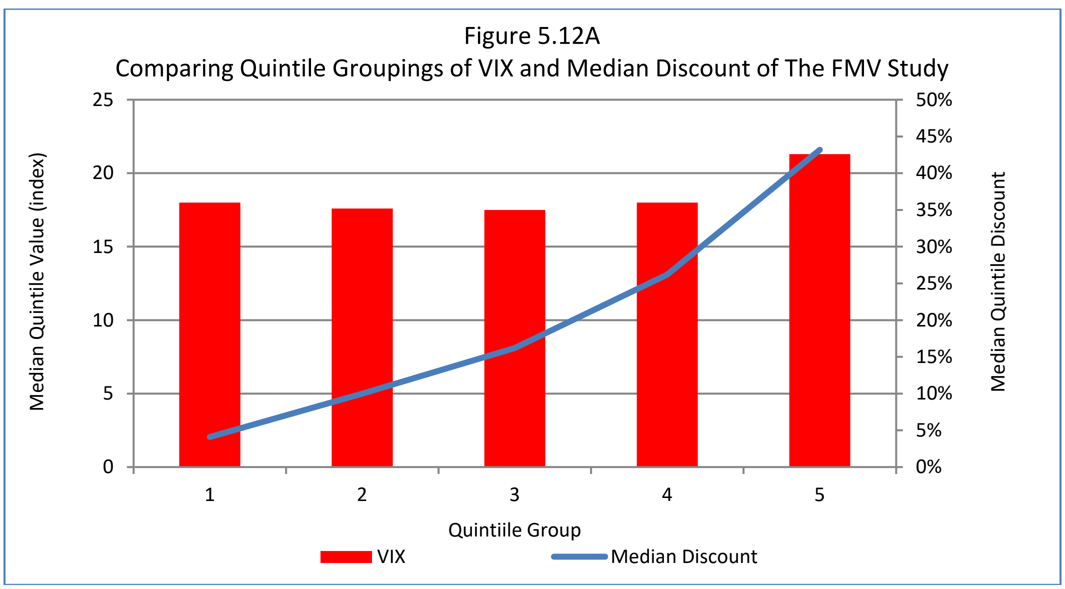


- The R-square of correlation of the quintile groupings of median discounts and price volatility is 98.3%, as shown in Figures 5.11A and 5.11B. Consistent with general expectations the regression line of Figure 5.11B is positive, meaning that discounts increase as volatility increases. But the regression formula results in 0% discounts for businesses with price volatility of 59% or less and 100% discounts for those with price volatility of 215% or more.





- The R-square of correlation of the quintile groupings of median discounts and the volatility index (VIX) is 71.0%, as shown in Figures 5.12A and 5.12B. Consistent with general expectations the regression line of Figure 5.12B is positive, meaning that discounts increase as volatility increases. But the regression formula results in 0% discounts when the VIX is 16.25 or less and 100% discounts when it is 30.5 or more.



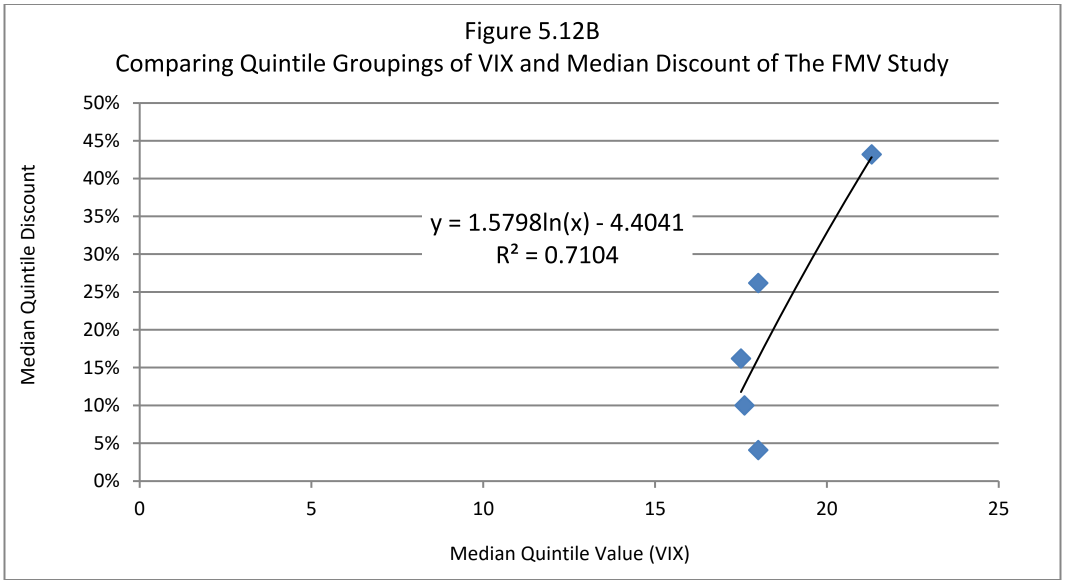


Table 5.5 summarizes the regression formula results described above. Although it can be questioned whether grouped data such as quintiles are statistically valid, it is clear that the Net Income variable is contradictory to DLOM (discounts go up as net income goes up), while the Net Profit Margin parameter is highly suspect (no discount if profit margin is greater than 3.725%). For the remaining variables practitioners should consider whether the ranges of implied discounts are reasonable.

Table 5.5
Range of Discounts Implied from Financial Characteristics of Transactions in the Stout Study

	Discount Range	
Independent Variable	0% Discount	100% Discount
Market Value	$227,500,000	$830,000
Revenues	$48,500,000	$710,000
Total Assets	$136,000,000	$260,000
Net Book Value	$62,000,000	$250,000
Market-to-Book Ratio	2.625x	27x
Net Income	($11,000,000)	$27,600,000
Net Profit Margin	3.725%	-96.00%
12-Month Price Volatility	59%	215%
Volatility Index (VIX)	30.50	16.25

Section 3 — The Discounts Reported in the Stout Restricted Stock Study Are Consistent with Past Changes in SEC Rule 144 Required Holding Periods

It was pointed out in Chapter 4 that the discounts reported in the Pluris® database are not intuitively consistent with changes in SEC Rule 144 required holding periods; average discounts should decrease as required holding periods decrease. However, the discounts reported in The Stout Study are intuitively consistent:

- The discounts for transactions reported with 2-year holding periods average 22.47% (24.1% if zero and negative discounts are excluded);
- The discounts for transactions reported with 1-year holding periods average 19.1% (21.9% if zero and negative discounts are excluded); and
- The discounts for transactions reported with 6-month holding periods average 14.3% (15.9% if zero and negative discounts are excluded).

Section 4 — How the Stout Restricted Stock Study Discounts Correlate with the Other Metrics Reported in the Database

A more robust analysis occurs if the detailed data reported in The Stout Study is analyzed without the quintile grouping approach employed by Stout Table 5.6 reports the results of linear and logarithmic regressions of the principal independent variables reported for the 769 transactions in The Stout Study. The strongest statistical relationship with restricted stock discounts is shown by annual price volatility. That relationship has R-squares of correlation of 10.6% and 15.4% based on the linear and logarithmic regressions, respectively.[86] It makes sense that negotiated restricted stock discounts are affected logarithmically by changes in price volatility. In other words, the changes in discounts and price volatility are not likely to represent a straight line.

[86] Price per share shows the next strongest relationship with a 13.1% logarithmic R-square of correlation. Intuition does not explain why share price would affect the discount. The next highest logarithmic R-squares of correlation are shown by the total liabilities, gross placement amount, and market value variables. It makes intuitive sense that discounts would vary with these variables because they have risk implications not present with price per share.

Table 5.6
Summary of Detailed Regression Analyses of The Stout Study

	Number of Transactions	Linear R-Square	Linear Slope	Logarithmic R-Square
Block Size	769	0.02230	0.31320	0.00440
Book Value	769	0.01440	0.00000	negatives
Dividend Yield	none	none	none	none
EBIT	733	0.00140	0.00000	negatives
EBIT Margin	681	0.00740	-0.00030	negatives
EBITDA	733	0.00170	0.00000	negatives
Gross Placement Amount	769	0.01210	0.00000	0.08250
Market Value	769	0.01680	0.00000	0.07550
MTB Ratio	769	0.03030	-0.00003	negatives
Net Income from Continuing Operations	769	0.00020	0.00000	negatives
Net Profit Margin	717	0.00700	-0.00020	negatives
Pretax Income	769	0.00040	0.00000	negatives
Price per Share	769	0.03830	-0.00270	0.13100
Prior Year Dividends per Share	769	0.01580	-0.06440	zeros
Retained Earnings	768	0.01240	0.00000	negatives
Shares Outstanding	769	0.00920	0.00000	0.02110
Shares Placed	769	0.00007	0.00000	0.00280
Shares Placed to Volume	769	0.01260	0.00009	zeros
Total Assets	769	0.00650	0.00000	zeros
Total Current Assets	716	0.00140	0.00000	zeros
Total Current Liabilities	717	0.00120	0.00000	zeros
Total Interest Bearing Debt	768	0.00370	0.00000	zeros
Total Liabilities	222	0.00740	0.00000	0.08710
Total Revenues	769	0.00450	0.00000	zeros
Transaction Day Close	769	0.00170	-0.00050	0.01420
VIX	695	0.00100	0.00080	0.00210
VIX 1Month	699	0.00090	0.00080	0.00190
VIX 3Month	696	0.00130	0.00090	0.00240
Volatility	740	**0.10630**	**0.06040**	**0.15350**
Volume	769	0.00050	0.00000	zeros
Volume to Shares Outstanding	769	0.01810	0.06150	zeros
Z Score	632	0.00610	0.00001	negatives

The Stout DLOM calculator relies on seven variables to benchmark DLOM: market value of equity, total revenues, total assets, shareholders' equity ("book value"), market-to-book ratio, net profit margin, and 12-month price volatility. Of the 769 transactions comprising The Stout Study, 217 had positive values for all seven of the variables used by the Stout calculator. Table 5.7 summarizes certain statistical characteristics of the 217 transactions according each of the seven calculator variables. Additionally, Table 5.7 presents the results of linear regressions of the transaction issuers' financial characteristics (i.e., the variables) and the corresponding transaction discounts. All of the variables showed logical relationships with discounts.

Table 5.7

Statistical Attributes of 217 Stout Restricted Stock Transactions with Positive Discounts and Positive Financial Parameters

	Transaction Discount	Market Value	Total Revenues	Total Assets	Book Value	MTB Ratio	Net Profit Margin	Volatility
Average ($000)	18.2%	$255,014	$239,082	$814,147	$123,713	5.6	12.5%	67.0%
Std Dev ($000)	15.1%	$622,326	$1,281,950	$3,081,418	$359,788	12.4	37.6%	43.7%
Coefficient of Variation	0.8	2.4	5.4	3.8	2.9	2.2	3.0	0.7
Count	217	217	217	217	217	217	217	217
Standard Error ($000)	1.0%	$42,246	$87,024	$209,180	$24,424	0.8	2.5%	3.0%
95% Confidence Interval								
• High ($000)	20.2%	$337,816	$409,650	$1,224,141	$171,585	7.3	17.5%	72.8%
• Low ($000)	16.2%	$172,211	$68,514	$404,154	$75,842	4.0	7.5%	61.2%
R-square of Linear Correlation with Discounts	N/A	1.81%	1.02%	0.90%	2.32%	4.25%	0.62%	22.92%
Direction of Slope	N/A	Negative	Negative	Negative	Negative	Positive	Negative	Positive
Slope logic	N/A	Logical	Logical	Logical	Logical	Logical	Logical	Logical

The 217 transactions comprising Table 5.7 were tested for statistical significance. Price volatility and market-to-book ratio have the highest linear R-squares of correlation with transaction discounts at 22.92% and 4.25%, respectively. Regression analysis of the group of transactions shows an overall 27.6% R-square of correlation. But Table 5.8 shows that only market-to-book ratio and price volatility are statistically significant with t-Stats greater than 2.0 and P-values less than 5%. Price volatility shows much more statistical strength than market-to-book ratio. Removing price volatility from the group reduced the R-square of correlation to 8.0%, with market-to-book value remaining the only statistically significant variable. The raw data for market value, total revenues, total assets book value, and net profit margin does not appear to offer a statistical basis for benchmarking DLOM.

Table 5.8
Multivariate Regression Analysis of 217 Stout Restricted Stock Transactions with Discounts and Financial Parameters Greater than Zero

Valuation Variable	*t Stat*	*P-value*	Significant?
Market Value	0.20904	0.83461	No
Total Revenues	-0.28434	0.77642	No
Total Assets	1.64831	0.10079	No
Book Value	-1.42682	0.15512	No
Market-to-Book Ratio	2.48965	0.01356	Yes
Net Profit Margin	-1.02218	0.30787	No
Price Volatility	7.53149	1.485E-12	Yes

The above analyses allow one to conclude with reasonable certainty that negotiators consider the issuer's stock price volatility when negotiating restricted stock discounts. Therefore, a reliable method for estimating discounts should focus on stock price volatility. Of course, it is unknown how negotiators estimate stock price volatility. It may be based on 12-month historical periods as analyzed here, other historical time periods, trends, or otherwise. But since investing is a matter of future expectations, it is reasonable to assume that negotiators of restricted stock prices are interested in the price risks associated with an expected holding period for their investment. It seems clear however that DLOM conclusions should be based primarily on estimated price risk over a period of illiquidity.

The Stout Study provides two data fields that are helpful in further analyzing the correlation of price volatility and discounts. Those are the Rule 144 holding period and the registration rights applicable to the restricted stock transaction. The Rule 144 information allows better consideration of the relationship of discounts to both time and price volatility. The registration rights information allows better consideration of the relationship of discounts to risk and liquidity. Table 5.9 summarizes the results of logarithmic regressions of price volatility versus different combinations of the time and liquidity-based information. Whereas the R-square of correlation for all 679 transactions was 21.1%, significant improvement was found in the correlation of the price volatility and the discounts for restricted stock transactions without registration rights and subject to a six-month Rule 144 holding period requirement. Those conditions applied to 24 restricted stock transactions that show a 35.0% R-square of correlation with the reported discounts. This result suggests that the discount negotiators were influenced to some extent by some combination of the period of illiquidity and registration rights.

Table 5.9

R-Squares of Logarithmic Correlation for Restricted Stock Transactions in the Stout Study with Both Positive Discounts and Price Volatility Reported

	Rule 144 Holding Period			
	All	2 Years	1 Year	6 Months
Rule 144 Holding Period				
R-square	21.1%	20.5%	22.8%	26.9%
Number of Transactions	679	232	318	129
With Registration Rights				
R-square	15.4%	15.8%	14.2%	23.1%
Number of Transactions	309	43	162	104
Without Registration Rights				
R-square	19.3%	21.7%	17.9%	35.0%
Number of Transactions	233	184	25	24
Unknown Registration Rights				
R-square	24.2%	22.9%	24.1%	N/A
Number of Transactions	137	5	131	1

Section 5 — The Stout DLOM Methodology

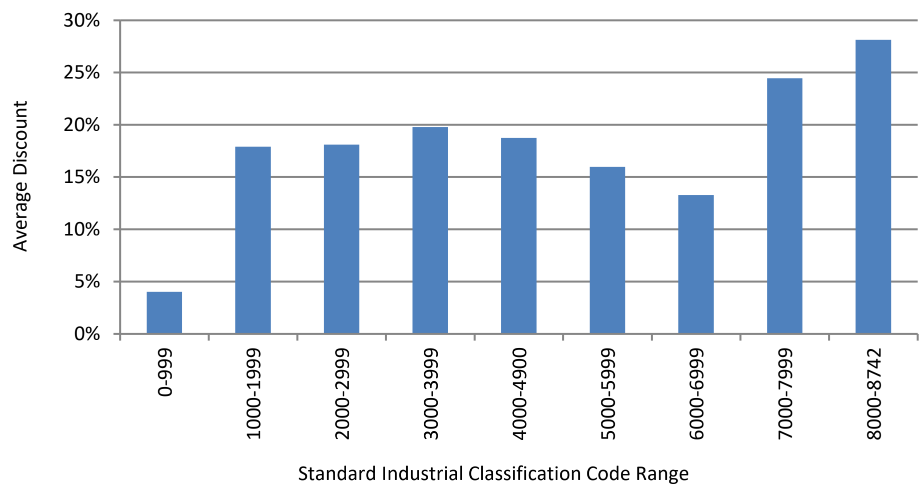
The Stout DLOM methodology is generally similar the Pluris® DLOM methodology, but with two principal differences. First, Stout bases its median values on a quintile division of its database transactions, while Pluris® bases its median values on a quartile division of its database transactions. Stout's® use of quintiles instead of quartiles provides its method a slight methodological advantage. Second, the variables used to benchmark valuation subjects are somewhat different. Both benchmark on total assets, total revenues, book equity, net profit margin, and market-to-book value ratio. The Stout method also benchmarks on market value of equity and price volatility, while the Pluris® method benchmarks on EBITDA, net income, and enterprise value. Stout's® use of price volatility as a benchmarking variable provides it with a statistically significant metric that the Pluris® method does not employ (except as a user-defined variable).

Stout states that it "typically does not consider industry classification to be a significant determinant of DLOM."[87] Nor does the Pluris® DLOM calculator employ industry as a benchmark.

[87] "Determining Discounts for Lack of Marketability – A companion Guide to The RMV Restrict Stock Study," FMV Opinions, Inc., 2015, page 22.

These omissions may be due to the dearth of transactions in specific SIC codes. But the average discounts of the Stout restricted stock transactions vary greatly by industry as Figure 5.13 shows. It is therefore illogical to exclude industry as a benchmarking variable.

Figure 5.13
Average Restricted Stock Discount by one-Digit SIC Code



The initial goal of the Stout DLOM methodology is to determine a restricted stock equivalent discount ("RSED").[88] There are five steps to this methodology:

1. The Stout database is sorted into five equal percentile groups (quintiles) for each variable and the median discount is computed for each group.[89]
2. The valuation subject's financial risk parameters are compared to the quintile group parameters to determine in which quintile segment the subject's parameter falls.[90]
3. The financial risk parameters of the valuation subject is "matched" to the quintile groups to obtain median discounts from the quintile segment deemed to be applicable to the subject company. The obtained median discounts are then averaged, with greater weight being given to quintile subsamples that have the greatest number of transactions.[91] The resulting average is considered to be the RSED.[92]

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[88] Id., at page 23.

[89] Id.

[90] Id.

[91] Id. Stout states that the weights are based on which factors tend to be the most important determinants of DLOM. Stout considers that the "key" variables are market value, total assets, shareholders' equity, and price volatility.91

4. The Stout methodology may then employ a “market volatility adjustment” to yield an adjusted RSED (“ARSED”) in the event that a valuation date occurs within a period of abnormally high market volatility. Stout states that transactions that occurred in periods of high market volatility tend to exhibit higher discounts.[93] Stout states that the RSED tends to underestimate the actual transaction discounts for high-VIX transactions.[94] Accordingly, Stout employs the “market volatility adjustment” for all one-year SEC Rule 144 holding period data high-VIX transactions.[95] This results in multiplicative adjustments of 1.16:1 for transactions in the 60th to 80th VIX percentile group and of 1.23:1 for transactions in the 80th to 100th VIX percentile group.[96] Stout uses implied adjustment factors for VIX index values greater than 32.9.[97] The VIX statistic utilized for this analysis is the trailing six-month average VIX as of the transaction date.[98] Stout advises that appraisers should also consider the possibility that a downward adjustment to the RSED may be appropriate during times of historically low stock market volatility.[99]
5. According to Stout, the ARSED represents the discount appropriate for a public company issuing restricted stock that will ultimately have access to a public trading market, and that an incremental private equity discount (“PED”) (generally a positive discount, but sometimes a negative discount) is needed to determine the discount appropriate for a privately held business.[100] The PED increment is derived by comparing the discount indications for large-block transactions with those for small-block transactions.[101]

   Stout states:

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[92] Id., at page 24.

[93] Id.

[94] Id.

[95] Id.

[96] Id.

[97] Id.

[98] Id.

[99] Id.

[100] Id. at page 25.

[101] Id.

> **[Key Point]** Unlike differing percentage minority interest in public companies, which have differing degrees of liquidity…,differing percentage minority interest in private entities generally have similar degrees of liquidity. Furthermore, the degree of liquidity of typical minority interests in private companies is most similar to the degree of liquidity of large blocks or restricted stock in public companies.[102]

Stout provides no substantiation for this statement, which is an essential foundation for its PED methodology. To the contrary, regression analysis of block size and transaction discounts as reported in The Stout Study yields extremely low R-squares of correlation. *See* Table 5.6.

According to Stout, large block transactions most closely resemble private equity and small-block transactions most resemble RSED.[103] As discussed in Chapter 4, these supposed similarities are questionable. Large stock blocks in privately held companies often represent controlling interests, which are generally considered to be more liquid and to require lower discounts than non-controlling interests. Why are large blocks of restricted stock not similarly more liquid than the smaller blocks? It seems illogical that smaller-block non-controlling interests would require smaller discounts than the larger-blocks that potentially represent some degree of control of the enterprise.

Setting aside what may be a seriously flawed PED methodology, it is reasonable that the liquidity discount applicable to an interest in a privately held business should be greater than its RSED. Some amount of PED is likely appropriate when valuing a privately held business.

Section 6 — Testing the Stout DLOM Methodology

The reliability of the Stout DLOM methodology is easily tested by (1) dividing into quintiles the seven Stout DLOM parameters (i.e., market value of equity, total revenues, total assets, shareholders' equity, market-to-book ratio, net profit margin, and 12-month price volatility); (2) determining the median discount for each quintile segment of each parameter; (3) matching the equivalent characteristics of the restricted stock issuers to the appropriate quintile segment; (4) averaging the resulting median discounts; and (5) comparing the DLOM results with the transactions discounts using linear regression.[104] Ideally the relationship is one-to-one. The

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[102] Id.

[103] Id.

[104] It is not necessary to weight the discounts for this exercise because the financial characteristics of the actual issuers should directly correspond to their negotiated discounts. Likewise, it is not necessary to adjust the resulting DLOM for market volatility conditions because

average ideally also would closely approximate the restricted stock discount of the issuer or, at least, for the population of transactions as a whole.

Figure 5.14 shows that the distribution of the quintile DLOMs is not consistent with the reported discounts of 638 underlying transactions with discounts greater than zero. While the distribution of discounts extends from less than 1% to 92%, the distribution of quintile DLOMs is bookended within the range of 11% to 27%. On a prima facie basis the quintile approach does not emulate the restricted stock discounts and is not a reliable way to estimate DLOMs.

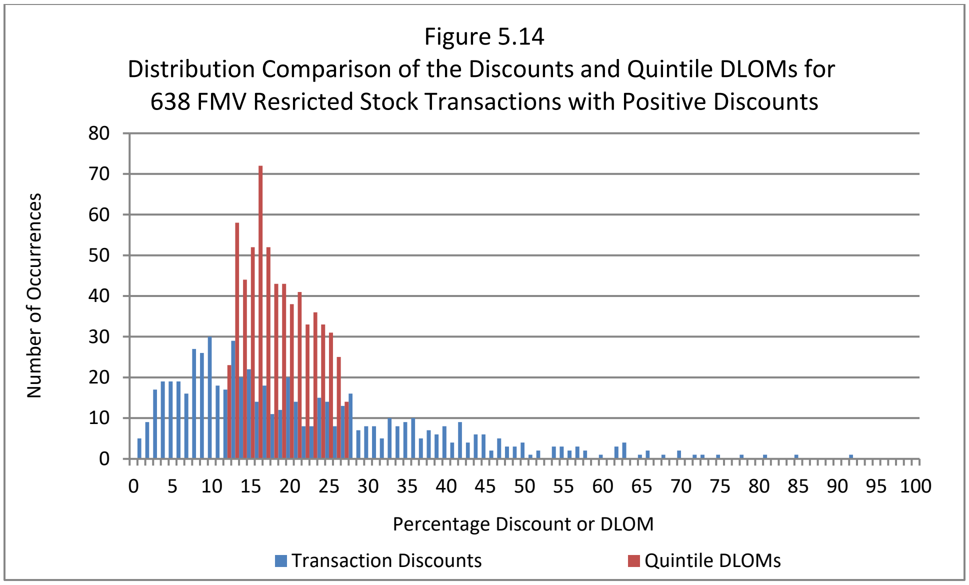

Table 5.10 shows different groups of restricted stock transactions extracted from The Stout Study. The transactions selected for analysis are those 638 for which The Stout Study reports positive discounts and values for each of the seven Stout DLOM calculator parameters (i.e., market value of equity; total revenues; total assets; shareholders' equity; market-to-book ratio; net profit margin; and 12-month price volatility). The groups are further differentiated by the Rule 144 time period and registration rights characteristics stated in the Study, and are ranked in descending order according to the number of transactions comprising each group.

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the discounts were negotiated contemporaneously with the prevailing conditions. Finally, a PED is unnecessary for this exercise since the issuers are public companies.

Table 5.10
Correlation of Median Quintile DLOMs with Positive Discount Transactions

Rule 144 Time Period	Registration Rights	Transaction Count	Average Transaction Discount	Average Stout DLOM	Regression Line Slope	Y-Axis Intercept	R-Square
All	All	638	21%	18%	1.926	-13.5%	26%
2 Years	No	178	24%	22%	1.714	-14.1%	29%
1 Year	Yes	159	17%	14%	2.420	-16.5%	30%
1 Year	Blank	121	27%	22%	2.067	-18.3%	25%
6 Months	Yes	94	15%	12%	4.230	-34.3%	15%
2 Years	Yes	40	24%	24%	3.391	-56.0%	45%
1 Year	No	23	25%	23%	1.436	-7.9%	22%
6 Months	No	18	17%	17%	2.155	-19.5%	60%
2 Years	Blank	5	23%	23%	1.000	0.0%	100%

A perfect linear regression has an x coefficient of 1.0, a y intercept of 0.0%, and an R-square of correlation of 100%. That result occurred with the five-transaction group with a two-year Rule 144 holding period and unknown registration rights. But this is merely a proof of the regression methodology, because a five-transaction group divided into quintiles should result in a perfect correlation as each DLOM equals its corresponding discount. Table 5.10 shows that none of the other transaction groups provides a reasonable corroboration of DLOMs with the actual discounts. For example, the best of the other correlations is the 18-transaction group with a six-month Rule 144 holding period and no registration rights. This group has a 60% R-square of correlation of DLOMs to discounts, and the average DLOM equals the average discount—17%. Those values are deceptive, however, because the x coefficient slope is 2.155-to-1 and the y intercept is -19.5%. Figure 5.15 demonstrates why this group does not offer a satisfactory justification of the Stout methodology despite its relatively high R-square of correlation. DLOMs based on this quintile group would always be greater than 19% but less than 56% despite that the transactions on which they are benchmarked may have discounts ranging from 0% to 100%.

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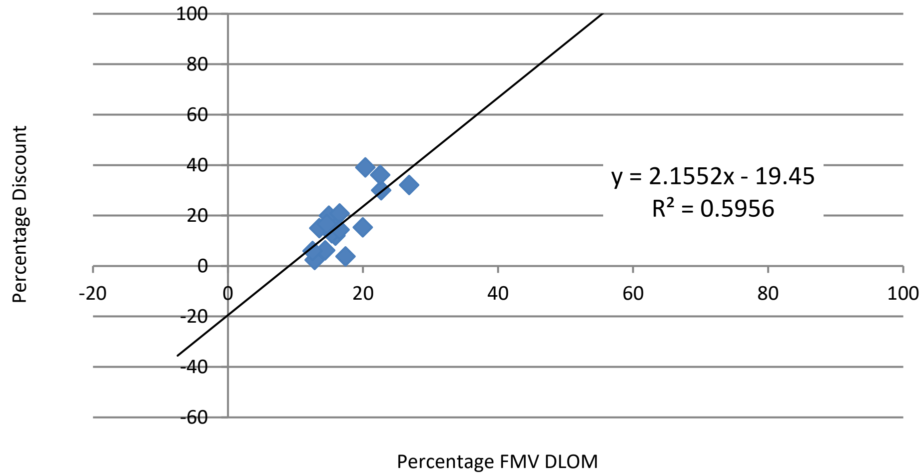
Figure 5.15
Correlation of DLOMs to Discounts for 18-Transaction Group



None of the other groupings reported in Table 5.10 offers a corroboration of DLOMs using the underlying restricted stock transactions. All have lower R-squares of correlation and unsatisfactory x coefficients and y intercepts.

Chapter 6

THE PRICE AND TIME VARIABLES THAT UNDERLIE DLOM

The business valuation concept of marketability deals with the liquidity of the ownership interest.[105] The ease and certainty with which an investor can quickly convert an investment to cash represent two very different variables. Ease of sale is a function of how quickly a sale can occur—the period of time it will take the seller to liquidate an investment. This period of time can vary greatly depending on the manner of sale. For example, liquidation sales can occur quickly and generally occur at lower prices, while orderly sales usually take longer to explore the marketplace of reasonable buyers and generally secure greater than liquidation prices. The time periods for private sales and public offerings also differ. In every instance, however, the "quickly" variable commences with a decision by the seller to initiate the sale process.

Price risk represents a lack of certainty that the expected price will be realized in an eventual sale. Price volatility is a way of quantifying the impact of the "certainty" variable during the period of time that it is being offered for sale. If market prices for similar investments fall dramatically while the marketplace is being explored, then the seller will have lost the opportunity to lock in the higher price that existed at the time the sell decision was made. Conversely, if the sale price is fixed for some reason (e.g., a listing agreement or a call price) and market prices for similar investments rise dramatically during the marketing period, the seller will have lost the opportunity to realize the increased value.

The time and price risk variables work together when determining an appropriate DLOM. Relative to immediately marketable investments, the value of illiquid investments must be discounted to reflect the uncertainties of the timing and realizable price of a sale. For example, assets may be subject to greater illiquidity during periods of market stress that would call for an increased DLOM. Transaction costs (particularly if similar costs are inherited by the buyer) may also impact the DLOM. These uncertainties reflected in business valuations are what DLOM should represent.

Logically, the economic costs of time and price uncertainties can be reduced to the price risk faced by an investor during the particular period of time that an illiquid investment is being offered for sale. In the market for publicly traded stocks, the volatility of stock prices represents risk. Investments with no price volatility have no DLOM, because they can be arbitraged to negate the risk of a period of restricted marketing—although perhaps with an interest cost. Conversely, volatile investments that are immediately marketable can be sold at the current price to avoid the risk of future volatility. It is different for investments in privately-held businesses.

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[105] Shannon P. Pratt and Alina V. Niculita, *Valuing a Business, 5th Edition: The Analysis and Appraisal of Closely Held Companies.* (McGraw-Hill, 2007), page 417.

The marketing period illiquidity experienced by the seller of a non-publicly traded business interest brings with it an economic cost reflective of the risk associated with the inability to realize gains and to avoid losses during that time period.[106] The longer that time period, the more the value of the business is exposed to adverse events in the marketplace and in the operations of the business, and the greater the DLOM that is required to equate the investment to an immediately liquid counterpart. Some or all of the economic cost associated with a period of illiquidity can be estimated using option pricing formulas such as Black-Scholes or the look-back formula developed by Francis A. Longstaff, Ph.D. in 2002,[107] which relies on estimates of price volatility (i.e., the *certainty* variable) and marketing time (i.e., the *quickly* variable).

Section 1 – Marketing Periods of Private Businesses Transactions

The marketing period for the private sale of a controlling interest in a business is seldom less than a few months, and can be much longer for a minority position in the business, as the following events occur:

- Drafting selling documents
- Developing a marketing strategy
- Implementing the marketing strategy
- Screening buyers
- Conducting site visits
- Assisting buyers in their analysis of the company and the interest being sold
- Drafting letters of intent
- Negotiating with the serious buyers
- Assisting buyers with due diligence
- Drafting the contract of sale
- Participating in arranging financing
- Actually closing the deal

The time periods of private sales of businesses were analyzed using 7,960 transactions from BV Resource's *DealStats®* database and 10,381 transactions from ValuSource's *BIZCOMPS®* database.[108] The *DealStats®* database included 32 transactions with zero or negative marketing periods (the dates may be transposed). Those transactions were excluded from the analyses below, leaving 7,928 *DealStats®* transactions for analysis. The population of

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106 Id.

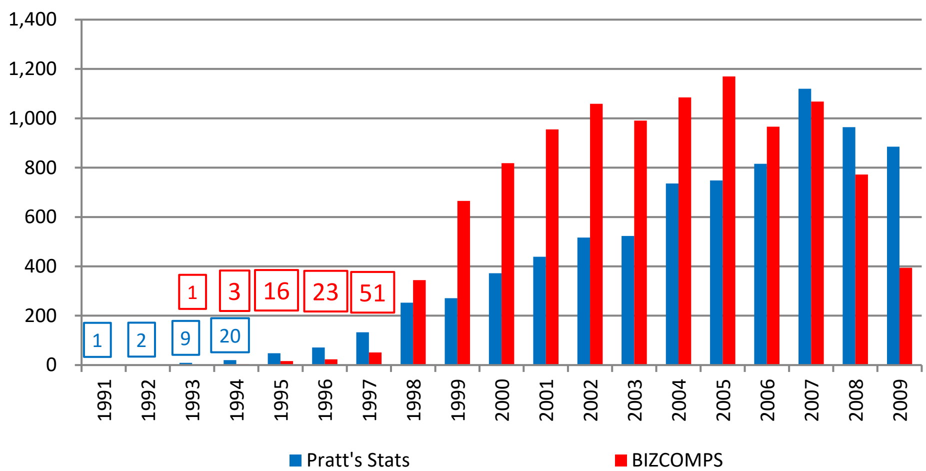
107 Francis A. Longstaff, "How Much Can Marketability Affect Security Values?", The Journal of Finance, Volume I, No. 5, December 1995.

108 Jack R. Sanders, CBA, CBI, CMEA, CVA is the collector and author of the *BIZCOMPS®* database.

the *DealStats*® transactions occurred from February 1992 through the end of 2011; the population of the *BIZCOMPS*® transactions occurred from March 1995 through the end of 2011.[109] For each transaction, these databases report an associated SIC code; sale initiation date; sale closing date; and asking price. Each *DealStats*® transaction also listed a market value of invested capital ("MVIC"). The two sets of transactions are heavily weighted to 1998 and later years. Figure 6.1 shows the distribution by year of the 7,928 *DealStats*® and 10,381 *BIZCOMPS*® transactions.

Readers will note the substantial declines in listings that occurred in years 2008 and 2009. These were the years of the "Great Recession," which likely explains the declines.

Figure 6.1
Listing Year Distribution of 7,928 *DealStats* and 10,381 *BIZCOMPS* Transactions



The average time that elapsed from the initial offering date to the closing date of these transactions is 211 days for the *DealStats*® transactions and 214 days for the *BIZCOMPS*® transactions. The standard deviation of the reported time periods is 208 days for the *DealStats*® transactions and 176 days for the *BIZCOMPS*® transactions.

Figure 6.2 shows the distribution of marketing periods of the population of *DealStats*® sales in 30-day increments.[110] The peak of the graph is 972 sale transactions that occurred from

[109] The transactions reported in the *DealStats*® and *BIZCOMPS*® databases reflect significant time period lags that can distort contemporaneous time period analysis by favoring sales that occurred quickly. Sales initiated after December 31, 2009, were excluded from both databases to avoid skewing the analyses herein with only short period sales in the years after 2009. Obviously, years have passed since the 2011 cutoff of the transactional data presented herein. The additional transactions subsequently reported in the *DealStats*® and *BIZCOMPS*® databases can be expected to have some effect on the results being discussed, and will be analyzed in later updates of this research

[110] Sixty-one transactions with marketing periods greater than 1,080 days were aggregated for presentation purposes.

61 to 90 days after being listed for sale. The 972 sales represent 12.3% of the population. One standard deviation to the right of the mean encompasses marketing periods up to 420 days, which is 88.5% of the population.

Figure 6.2
Distribution of Selling Time of 7,928 *DealStats* Transactions

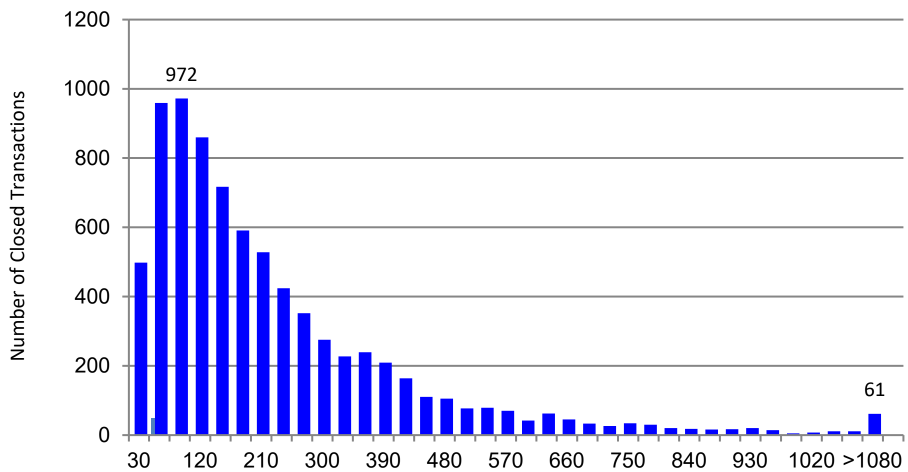
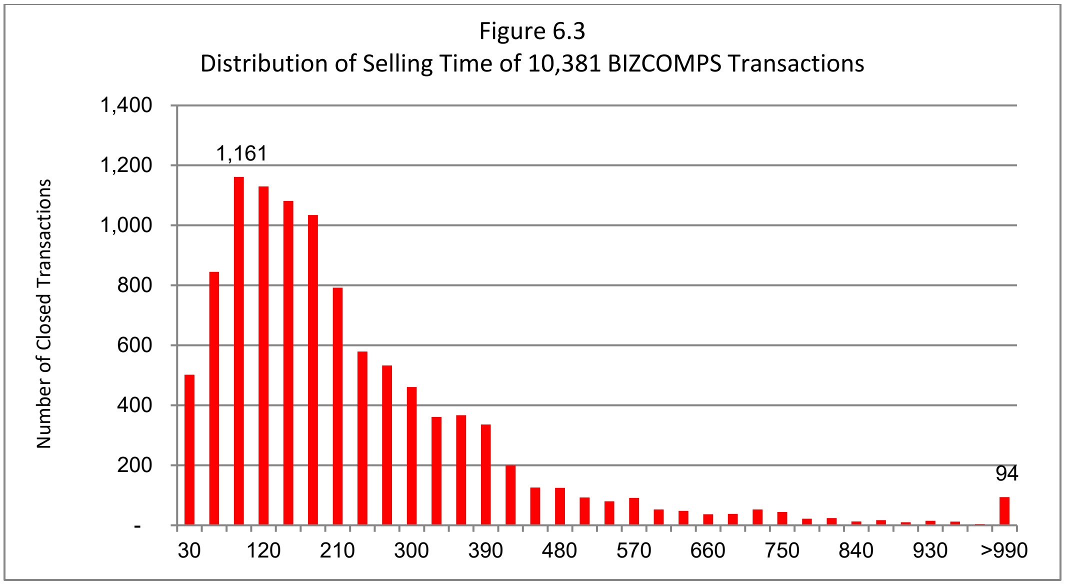


Figure 6.3 shows the distribution of marketing periods of the population of *BIZCOMPS®* transactions in 30-day increments.[111] The peak of the graph is 1,161 sale transactions that occurred from 61 to 90 days to sell. The 1,161 sales represent 11.2% of the database. One standard deviation to the right of the mean encompasses marketing periods of up to 390 days, which is 88.5% of the population. The transactions reported by *BIZCOMPS®* occurred faster on average than those reported in by *DealStats®*.

[111] Ninety-four transactions with marketing periods greater than 990 days were aggregated for presentation purposes.



Section 1.A Industry Variations in Selling Time

Separating the *DealStats*® and *BIZCOMPS*® transactions into broad industry groupings represented by ten two-digit SIC code divisions shows significant variation of selling periods between industries. *See* Figure 6.4.

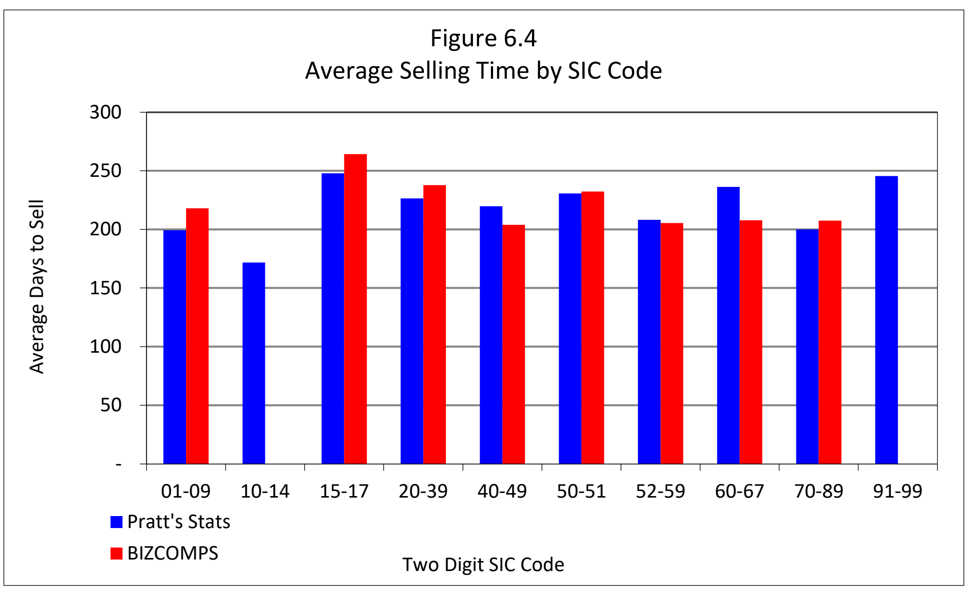


Table 6.1 presents the number of transactions, average selling time in days, and standard deviation of the selling times by two-digit SIC code. The spread between the fastest average selling and slowest average selling industry groups is 76 days in the *DealStats*® database and 60 days in the *BIZCOMPS*® database.

Table 6.1
Selling Time of Private Company Sales by Two-Digit Industry Classification

SIC Code Range	SIC Group	*DealStats®* Number of Sale Transactions	*DealStats®* Average Selling Time in Days	*DealStats®* Standard Deviation	*BIZCOMPS®* Number of Sale Transactions	*BIZCOMPS®* Average Selling Time in Days	*BIZCOMPS®* Standard Deviation
01-09	Agriculture, forestry, and fishing	235	199	201	310	218	182
10-14	Mining	9	172	127	-	n/a	-
15-17	Construction	375	248	260	503	264	198
20-39	Manufacturing	918	226	212	1,060	238	197
40-49	Transportation, communications, electric, gas, and sanitary services	245	220	225	330	204	172
50-51	Wholesale trade	502	231	241	579	232	205
52-59	Retail trade	2,833	208	204	4,053	205	166
60-67	Finance, insurance, and real estate	136	236	249	217	208	193
70-89	Services	2,673	200	193	3,329	207	168
91-99	Public administration	2	246	257	none	n/a	-
	All industries	7,928	211	208	10,381	214	176

The construction industry group had the longest average marketing period in both the *DealStats®* and *BIZCOMPS®* databases: 248 days and 264 days, respectively. The finance/insurance/real estate and manufacturing industry groups also had marketing periods longer than the mean in both databases.[112]

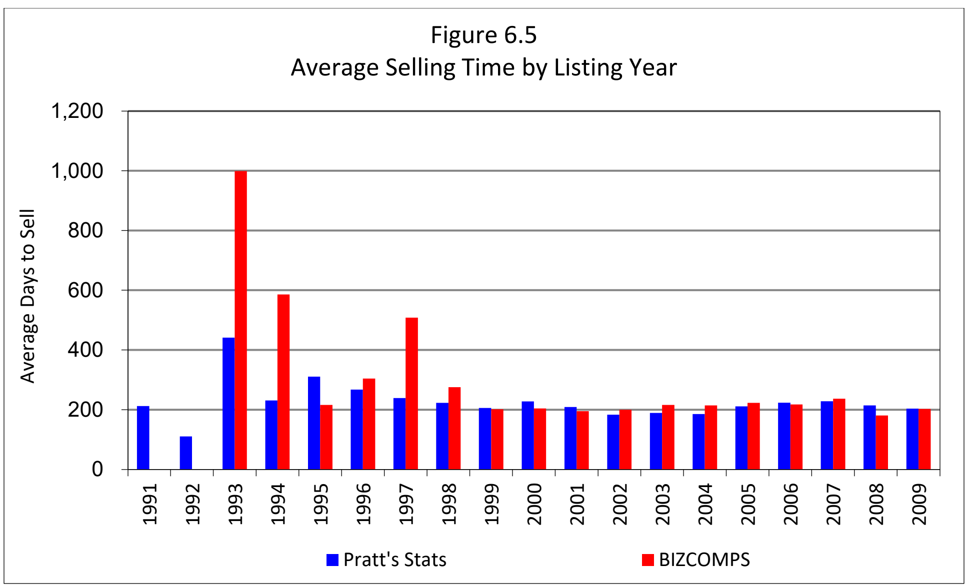
Businesses reported in the mining industry sold relatively quickly in an average of 172 days, but is based on only nine translations in the *DealStats®* database. The *BIZCOMPS®* database contains no mining industry transactions. The retail and services industry groups also had marketing periods shorter than the mean in both databases. The *DealStats®* and *BIZCOMPS®* databases had inconsistent results relative to the mean for the agriculture/forestry/fishing, transportation/communications/electric/gas/sanitary services, and finance/insurance/real estate industry groups.

The above results show that average marketing periods are materially different for businesses operating in different industries. The widely varying standard deviations of marketing periods add to the differences that can be expected when comparing one business to another.

[112] The public administration industry group is ignored since it represents the sale of just two businesses.

Section 1.B Selling Time from Year-to-Year

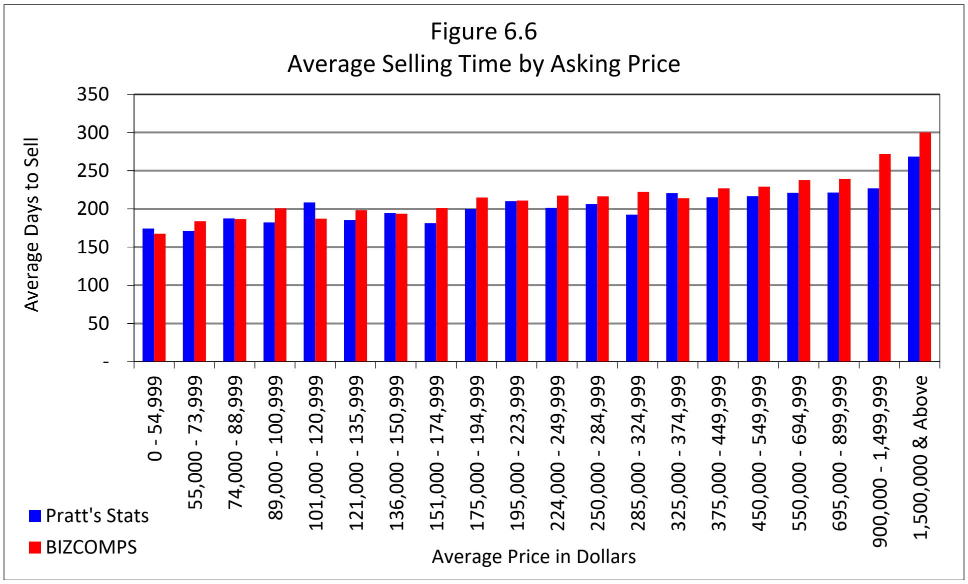
Figure 6.5 shows the average selling time of the *DealStats*® and *BIZCOMPS*® transactions when they are divided according to the year in which the businesses were listed for sale. When considering Figure 6.5, keep in mind that the two databases have very few transactions prior to 1998, which may affect the averages in those years. *See* Figure 6.1. Figure 6.5 shows a reduction in average selling times in 2008 and 2009, the years of the "Great Recession." *DealStats*® transactions with listing dates in 2007 took an average 228 days to sell, but those listed in 2008 and 2009 took, respectively, 214 and 204 days on average to sell. *BIZCOMPS*® transactions with listing dates in 2007 took an average 237 days to sell, but those listed in 2008 and 2009 took, respectively, 181 and 202 days on average to sell. These results, seemingly contrary to "Great Recession" intuition, may be an indication that the businesses listed for sale in 2008 and 2009 were sold under duress, were more desirable than historically because fewer troubled business may have been offered for sale, or may be a reflection of reduced supply. The private sale databases do not lend themselves to a ready determination of the cause of the shortened selling periods.



Section 1.C The Effect of Asking Price on Selling Times

The *DealStats*® and *BIZCOMPS*® databases provide the asking prices for most of the reported transactions. However, 565 *DealStats*® transactions and one *BIZCOMPS*® transaction have no asking price reported. These transactions were excluded from the asking price analysis. The range of asking prices of the resulting transaction populations were from $3,456 to

$70,000,000 for *DealStats*® and from $15,000 to $35,000,000[113] for *BIZCOMPS*®. Dividing the transactions into asking price groupings of roughly equivalent counts shows that the average number of days to sell a privately held business generally increases as the asking price increases. *See* Figure 6.6.



Section1.D The Influence of Seasonality on Selling Time

The time of year in which a business is listed for sale seems to make a difference in the marketing period length. Figure 6.7 shows the selling times of businesses according to the month they were listed for sale.

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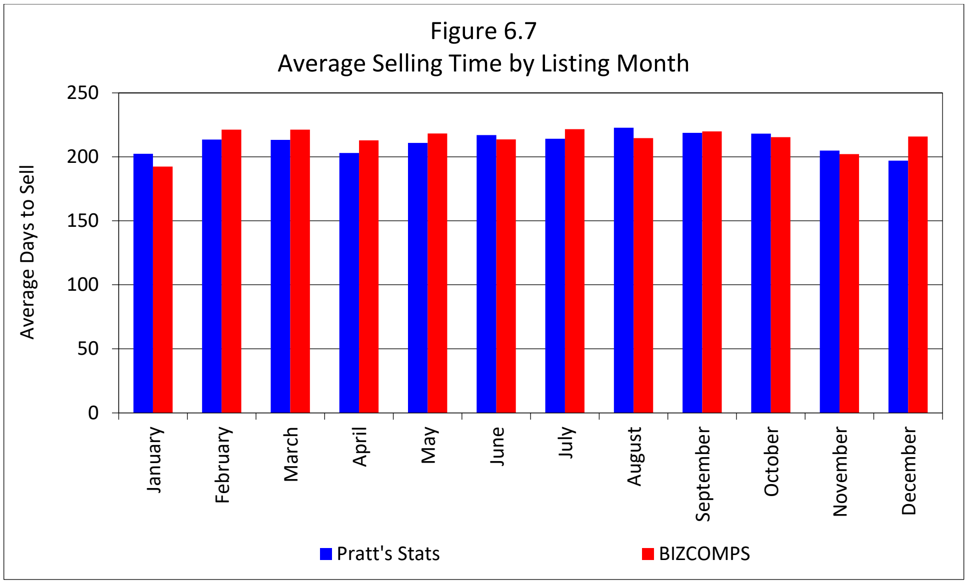
[113] One transaction had an asking price of $0, and was excluded from this calculation and Graph 6.



On average, sale transactions in the *DealStats*® database listed in August took the longest time to sell, with a mean of 223 days. Sales transactions in the *BIZCOMPS*® database originally listed in July took the longest time to sell, with a mean of 222 days. Listings in March for the *DealStats*® database and February for the *BIZCOMPS*® database had the highest variation of selling time. The months with the shortest marketing periods based on listing date were December, January, April, and November for *DealStats*® database (averaging 197, 202, 203, and 205 days, respectively), and were January and November for the *BIZCOMPS*® database (averaging 192 and 202 days, respectively). Possible explanations for the differences among the months are proximity to yearend numbers for November, December, and January listings, and proximity to completion of tax filings for April listings. Such proximity tends to offer buyers enhanced transparency through timelier financial reporting.

Section 2 – The Registration Periods of Public Offerings

The issuers of restricted stock transactions are, by definition, publicly traded companies. Consequently, a proper analysis of the time-period risks that accompany investments in restricted stocks should consider the probability and timing of eventual registration of restricted stock offerings. Vianello Forensic Consulting LLC, ("VFC") studied the probability and timing of obtaining registration approval from the SEC for 19,760 Form S-1 filings over the roughly 21.6 years from March 8, 1994, to October 19, 2015. Form S-1 is used to apply for securities registration with the SEC. VFC then determined the type of security for which registration was requested, and whether the application was approved, withdrawn, or is still pending. The S-1 filings were classified as equity, notes, or a mixture of equity and notes, to the extent possible.

4,761 of the applications could not be readily classified as equity, notes, or a mixture of those two types, or had no SIC code disclosed. These were left unclassified.

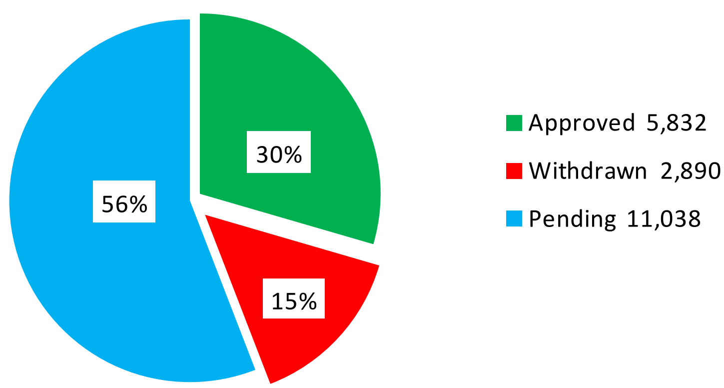
Table 6.2
Form S-1 Approval Status by Type of Security

	Approved	Withdrawn	Pending	Total
Equity	5,157	2,385	5,632	13,174
Notes	229	80	649	958
Mixed Equity & Notes	200	125	542	867
Unclassified or No SIC Code[114]	246	300	4,215	4,761
	5,832	2,890	11,038	19,760

Source: Vianello Forensic Consulting, LLC "SEC Time Period Study"

Table 6.2 shows that not all of the 19,760 S-1 filings resulted in approved offerings. Only 5,832 of the applications were approved as of October 19, 2015, representing 30% of the 19,760 applications. A total of 2,890 registration applications were withdrawn—15%. And a surprising 11,038 applications (56%) were still pending as of October 19, 2015.

Figure 6.8
SEC Registration Filing Status



[114] Securities were classified according to the Form S-1 tables reported by the SEC. VFC used an automated process to collect the issuers' SIC codes. If the table could not be found then the securities were not classified. Separately, no SIC code was reported by the SEC for a small number of registrants. Manual investigation indicates that the automated process performed reliably. The large number of unclassified pending transactions may become classified as they move through the SEC approval process to be captured in later updates of this research.

The primary purpose of this chapter section is to consider the time probability distribution of successful S-1 registrations of equity securities; no analysis of the "pending" applications is made other than to classify them by type of security and to consider them by age. It may be of some other analytical interest that much of the "pending" applications is comprised of notes and mixed securities. Also, many of the pending applications are old. Whether they are forgotten/abandoned filings or some kind of "shelf registrations" was not explored.

Considering only the "approved" and "withdrawn" equity applications indicates that about 32% of equity registration filings were withdrawn for some reason. It can therefore be said that a third of all equity registration filings fail.

The present analytical interest is first with the 5,157 approved equity registration applications for the purposes of analyzing restricted stock discounts and predictive DLOM modeling, and second with the 2,385 withdrawn equity registration applications for comparative purposes.

The S-1 filings of the approved registrations were compared to the companion Forms 424B that priced the offerings for sale. The difference between the S-1 filing date and the 424B approval date provided the elapsed time for SEC processing.

Figure 6.9 is a chronological presentation of the time required to obtain SEC approval for equity security registrations over the 1994 through 2015 time period. It took an average of 97 days to obtain registration approval. But Figure 6.9 shows that the SEC processing time was much greater than average for applications filed during the first four years of the Obama administration, January 2009 through December 2012 than before and after. These facts disclose the importance of timing, prevailing economic conditions, and maybe governmental administration on the time required to register a security.

Figure 6.9
Time Periods for Equity Security Registration Approval by the SEC
Source: Vianello Forensic Consulting, LLC "SEC Time Period Study"

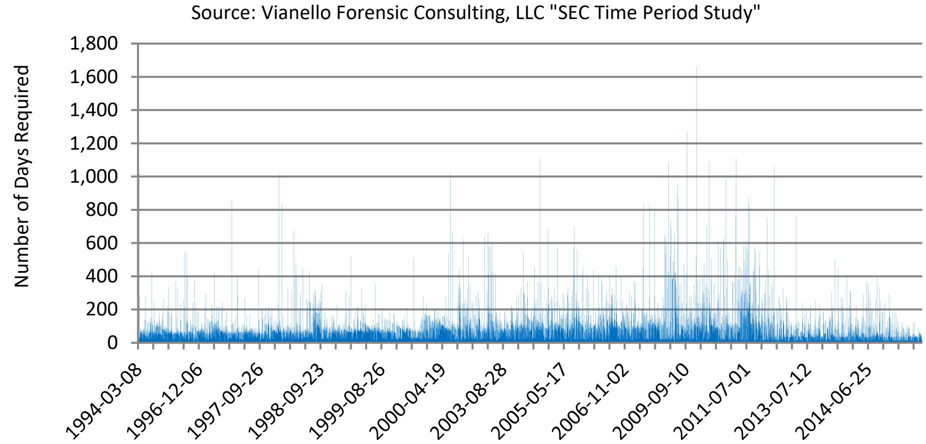
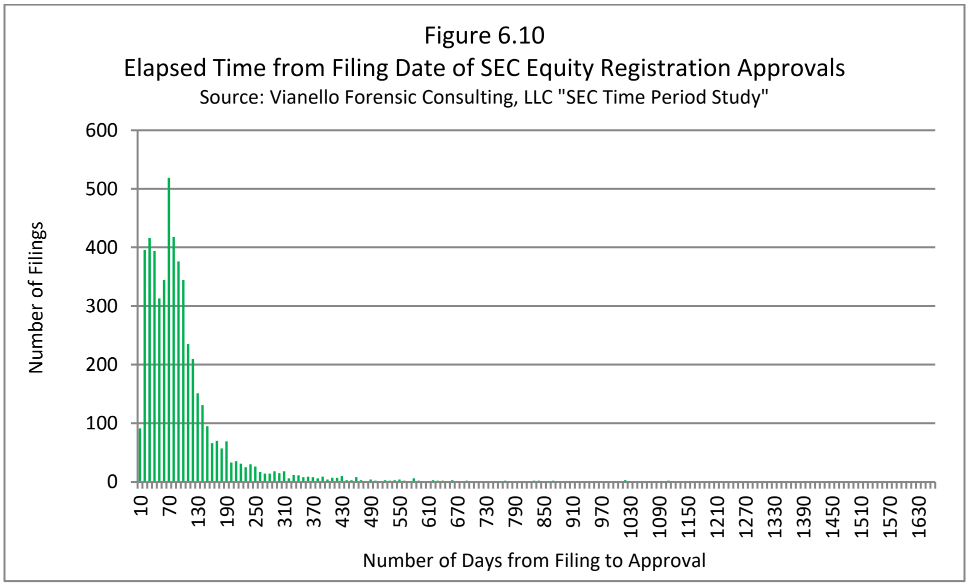
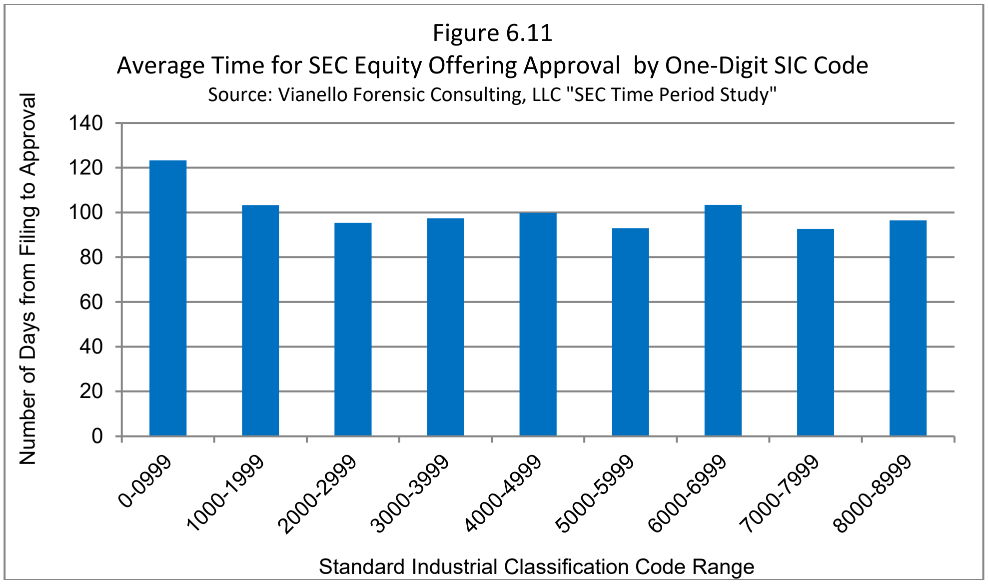


Figure 6.10 is a histogram of the time period frequencies. As stated, the average time required to process an approved equity security offering during the 1994 to 2015 study period was 97 days. But the underlying data is not flat and instead distributes log-normally as Figure 6.10 shows. Most frequently it took 73 days to process an approved S-1 application for an equity offering. Half of the applications were approved within 63 days, and half took much longer. One application required 1,659 days (4.5 years) to be approved.



It is also noteworthy that the time required to process approved S-1 equity security applications differed by industry. Figure 6.11 compares the average time across the nine broadest SIC codes present among the approved applications. While the 5000 and 7000 series codes average 93 days of processing time for approval, the 0-0999 series required 123 days—32% more time.



SEC processing times also deviate from average times differently by industry. Comparing the coefficients of variation in Table 6.3 reveals that the 5000 SIC code series is 86% more dispersed than the 0-0999 series. These variations show up in significant differences in the minimum and maximum days of processing time, standard error, and 95% confidence intervals. While the 95% confidence interval of all 5,157 approved S-1 equity registration applications was just 6 days, it was 25 days for the 1000 SIC code series and 98 days for the 0-0999 series.

Table 6.3
Approved SEC Equity Filings

	0-0999	1000-1999	2000-2999	3000-3999	4000-4999	5000-5999	6000-6999	7000-7999	8000-8999	All
Number of Filings	13	271	788	959	421	436	711	1,256	302	5,157
Average Days to Approval	123	103	95	97	100	93	103	93	96	97
Minimum Days	40	7	6	4	13	1	2	3	10	1
Maximum Days	298	1,017	1,091	1054	658	1,659	1,107	1,267	651	1,659
Standard Deviation	90	107	117	107	94	122	107	93	94	105
Coefficient of Variation	0.7	1.0	1.2	1.1	0.9	1.3	1.0	1.0	1.0	1.1
Standard Error	25	6	4	3	5	6	4	3	5	1
95% Confidence High Days	172	116	103	104	109	104	111	98	107	100
95% Confidence Low Days	74	91	87	91	91	82	95	87	86	94

Source: Vianello Forensic Consulting, LLC "SEC Time Period Study"

Much longer time period variations exist within the group of withdrawn S-1 equity registration applications. These similar but different results are presented with Figures 6.12 and 6.13 and Table 6.4. The characteristics of withdrawn applications may affect DLOM conclusions, particularly regarding initial public offerings. The DLOM applicable to the risk that an offering may fail for some reason is logically greater than the DLOM applicable to successful offerings.

Figure 6.12
Elapsed Time from Filing Date of SEC Equity Registration Withdrawals
Source: Vianello Forensic Consulting, LLC "SEC Time Period Study"

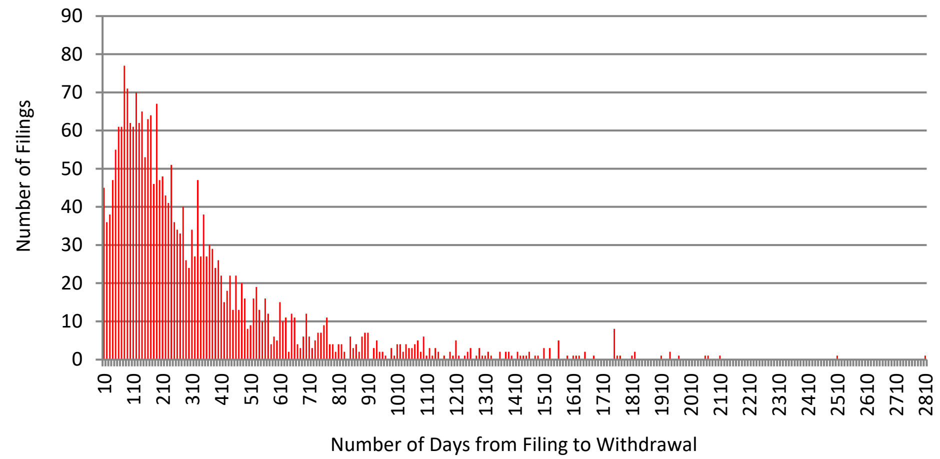


Figure 6.13
Average Time for Withdrawal from SEC Registration
Source: Vianello Forensic Consulting, LLC "SEC Time Period Study"

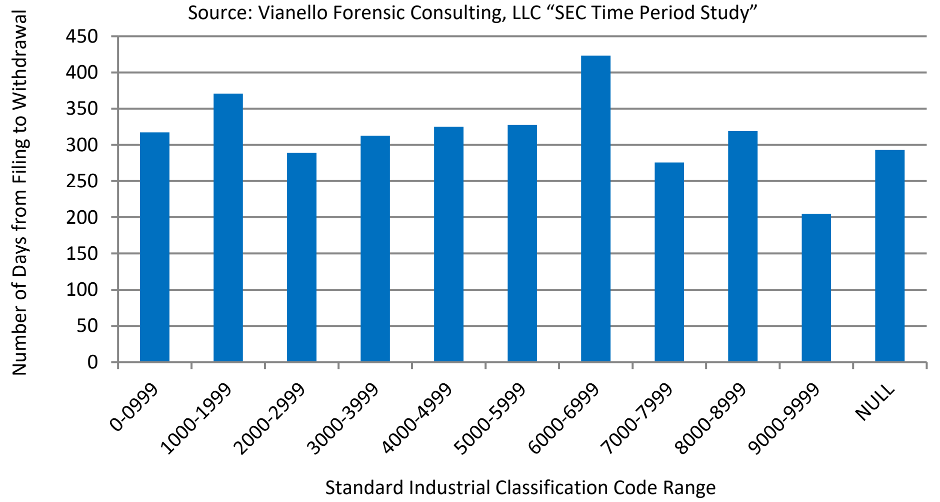


Table 6.4
Withdrawn SEC Equity Filings

	0-0999	1000-1999	2000-2999	3000-3999	4000-4999	5000-5999	6000-6999	7000-7999	8000-8999	9000-9999	All
Number of Filings	10	199	336	455	207	204	266	550	155	3	2,385
Average Time to Withdrawal	317	371	289	313	325	327	423	276	319	205	321
Minimum Days	39	0	0	5	0	0	1	0	1	56	-
Maximum Days	883	1,932	2,507	2,051	2,809	1,273	1,964	1,765	2,062	447	2,809
Standard Deviation	238	364	310	312	344	266	417	266	326	211	322
Coefficient of Variation	0.8	1.0	1.1	1.0	1.1	0.8	1.0	1.0	1.0	1.0	1.0
Standard Error	75	26	17	15	24	19	26	11	26	122	7
95% Confidence High Days	465	421	322	341	372	364	473	298	370	444	334
95% Confidence Low Days	169	320	256	284	278	291	373	253	268	0	308

Source: Vianello Forensic Consulting, LLC "SEC Time Period Study"

DLOMs calculated using option models require price volatility and time period assumptions. We sought to determine the correlation of probability-based Longstaff and Black-Scholes formula DLOMs with the observed discounts of restricted stock offerings using the issuer's stock price volatility for the twelve months preceding the stock sale date and the time periods for obtaining SEC approval for public stock offerings. This analysis used the mean and standard deviation of the issuer's stock price volatility and the mean and standard deviation of the SEC approval time for the SIC code corresponding to the issuer. We then calculated double probability DLOMs using the two option formulas. "Double probability" is discussed later in this chapter.

Table 6.5 starts with a group of 194 restricted stock transactions with corresponding SEC filings using the first digit of the issuers' four-digit SIC codes.[115] Table 6.5 reports that double probability DLOMs calculated using the VFC Longstaff methodology had an R-square of correlation of 16.45%, and that double probability DLOMs calculated using the VFC Black-Scholes methodology had an R-square of correlation of 20.51%.[116] Matching restricted stock transactions to SEC approval time periods using the first two digits of SIC codes reduced the number of transactions with corresponding SEC matches to 188. Table 6.5 shows little change in correlation for this group of transactions. The correlations improved further when transactions were matched to SEC approval time periods using three and four-digit SIC codes. A total of 118 restricted stock transactions matched on a four-digit basis. For these, the VFC Longstaff

[115] The 194-transaction group is a subset of the 200-transaction set presented in Chapter 8 at Table 8.1. Six transactions with a 9999 SIC code were excluded.

[116] The VFC Longstaff DLOM, VFC Black-Scholes DLOM, and double probability methodologies are discussed in Chapter 7.

methodology has an R-square of correlation of 26.94% and the VFC Black-Scholes methodology has an R-square of correlation of 31.98% with the issuers' restricted stock discounts.

Table 6.5 also shows that the quality of correlation improved. As the restrictions for matching the SIC codes of the restricted stock transactions to the SIC codes of approved SEC filings increased, the x coefficient of the VFC Longstaff regression line moved closer to 1.0 and the y intercept moved closer to zero. Likewise, the VFC Black-Scholes results improved with more specific SIC code matching, with the x coefficient of the VFC Black-Scholes regression moving closer to 2.0 and the y intercept moving closer to zero. It can therefore be said that DLOM reliability is enhanced when the valuation subject's industry is considered with as much specificity as possible.

Table 6.5

Correlations of DLOM and Discounts Improve with Better SIC Code Matching

						Linear Regressions v Transaction Discounts					
						Double Probability VFC Longstaff DLOM			Double Probability VFC Black-Scholes DLOM		
Number of Restricted Stock Transactions	Closing Date Range	SIC Code Digits Required for SEC Time Period Match	Number of SEC Approvals in the Issuers' SIC Codes	Transaction Discount	Registration Rights	Slope	Intercept	R-Square	Slope	Intercept	R-Square
194	2007-2014	1	4 or more	> 0%	DR, MR, NR, PB, No, Yes, and Unknown	0.7405	4.66%	16.45%	1.6660	3.94%	20.51%
188	2007-2014	2	4 or more	>0%	DR, MR, NR, PB, No, Yes, and Unknown	0.7167	5.38%	16.39%	1.6109	4.68%	20.30%
157	2007-2014	3	4 or more	>0%	DR, MR, NR, PB, No, Yes, and Unknown	0.8127	3.82%	20.93%	1.7944	3.38%	24.92%
118	2007-2014	4	4 or more	> 0%	DR, MR, NR, PB, No, Yes, and Unknown	0.8984	2.15%	26.94%	1.9796	1.61%	31.98%

Source: Vianello Forensic Consulting, LLC "SEC Time Period Study"

The Table 6.5 relationships are consistently strongly statistically significant at the 95% level of confidence, with t Stats well above 2.0 and P-values well under .05. *See* Table 6.5A. This means that we can reject the hypothesis that DLOMs calculated using the double probability VFC Longstaff and double probability VFC Black-Scholes methodologies do not correlate with the SIC codes of restricted stock issuers.

Table 6.5A

Statistical Significance of DLOM and Discounts as SIC Code Matching Improves

						Linear Regressions v Transaction Discounts			
						Double Probability VFC Longstaff DLOM		Double Probability VFC Black-Scholes DLOM	
Number of Restricted Stock Transactions	Closing Date Range	SIC Code Digits Required for SEC Time Period Match	Number of SEC Approvals in the Issuers' SIC Codes	Transaction Discount	Registration Rights	t Stat	P-value	t Stat	P-value
194	2007-2014	1	4 or more	> 0%	DR, MR, NR, PB, No, Yes, and Unknown	6.1	4.5E-09	7.0	3.5E-11
188	2007-2014	2	4 or more	>0%	DR, MR, NR, PB, No, Yes, and Unknown	6.0	8.3E-09	6.9	8.8E-11
157	2007-2014	3	4 or more	>0%	DR, MR, NR, PB, No, Yes, and Unknown	6.4	1.7E-09	7.2	2.8E-11
118	2007-2014	4	4 or more	> 0%	DR, MR, NR, PB, No, Yes, and Unknown	6.5	1.7E-09	7.4	2.5E-11

Source: Vianello Forensic Consulting, LLC "SEC Time Period Study"

Section 3 – Price Volatility

Investors have much less ability to control price risk than to control the time required for selling an illiquid asset. For example, a seller can influence the time it might take to sell a business by increasing or decreasing the asking price, having good financial reports to shorten due diligence periods, actively promoting the business, offering seller financing, etc. In contrast, price volatility occurs despite sellers' actions. Figure 6.14 shows the annual price volatility of the S&P 500 from January 2, 2008, to August 24, 2017, as measured by the VIX, the volatility index of the Chicago Board Options Exchange. The years of highest implied price volatility are those in which investors can be expected to have experienced increased difficulty locking in gains and avoiding losses.

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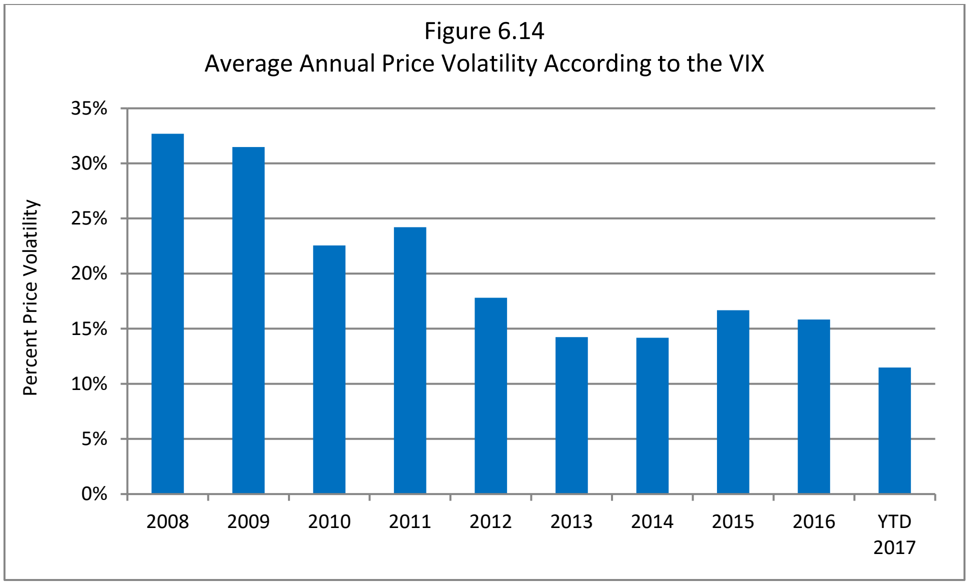
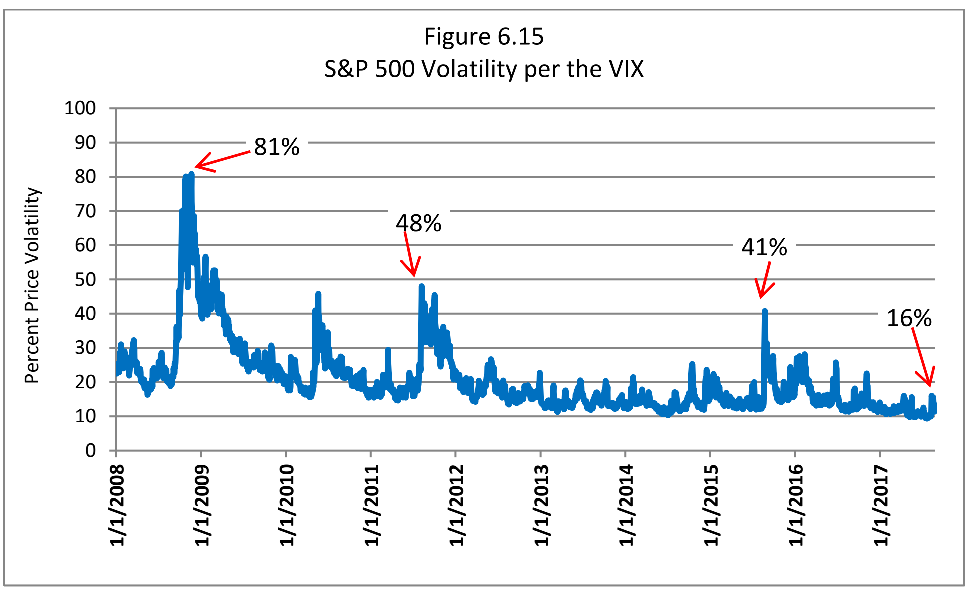


Stock market price volatility has been generally trending downward since January 1, 2008. Figure 6.14 shows that the VIX averaged 33% in 2008, declined to 14% in 2013 and 2014, and averaged 11% in 2017 to date. But Figure 6.15 shows that the downward trend of average annual price volatility has been sprinkled with periods of very high price volatility.



While the return volatility of the stock market is readily available, and the price volatility of publicly traded stocks is easily calculated from publicly available price data, practitioners often correctly observe that the price volatility of an interest in a privately owned business is not known. However, a reasonable estimate of the price volatility of a non-public company is easily made if the appraiser can identify at least one appropriate publicly traded company to use as a benchmark.[117] Alternatively, the practitioner may conclude that an index such as the S&P 500 or the VIX would be an appropriate price volatility surrogate, although consideration should be given to tendencies of broad indexes to negate the unsystematic risks of the individual stocks that comprise it, thereby understating the price risks of the underlying stocks. The average price volatilities of the index constituents may, therefore, be better measures of risk than the index.

Benchmarking choice is obviously a matter of considerable professional judgment. Practitioners who use the publicly traded guideline valuation method in their business valuations should use the same companies for price volatility estimation. A method of price volatility estimation for the privately held company might then be the annualized average stock price volatility for each of the guideline companies for an historic period of time considered predictive of the period of time expected to market and sell the interest being valued. Adjustments to the calculated price volatility may then be deemed appropriate.[118]

The very high volatility events shown in Figure 6.15 must be accounted for in a properly devised price volatility estimate. This can be done with probability analysis.[119] For example, the values shown in Figure 6.15 have a mean of 20.433% and a standard deviation of 9.977%, which, when graphed log-normally, distribute as per Figure 6.16.[120] The distribution discloses that the most frequently expected price volatility is 14.8% (the mode), and that half of all price volatility would be expected to be above and below 18.4% (the median). We see that 85% of all price volatility would be expected to occur below 29.4%. Basing a price volatility estimate on the full range of the log-normal curve accounts for the probability of all price volatility events displayed in Figure 6.15. In this case, the probability adjusted price volatility is 20.3%--a number that is less than the statistical average.

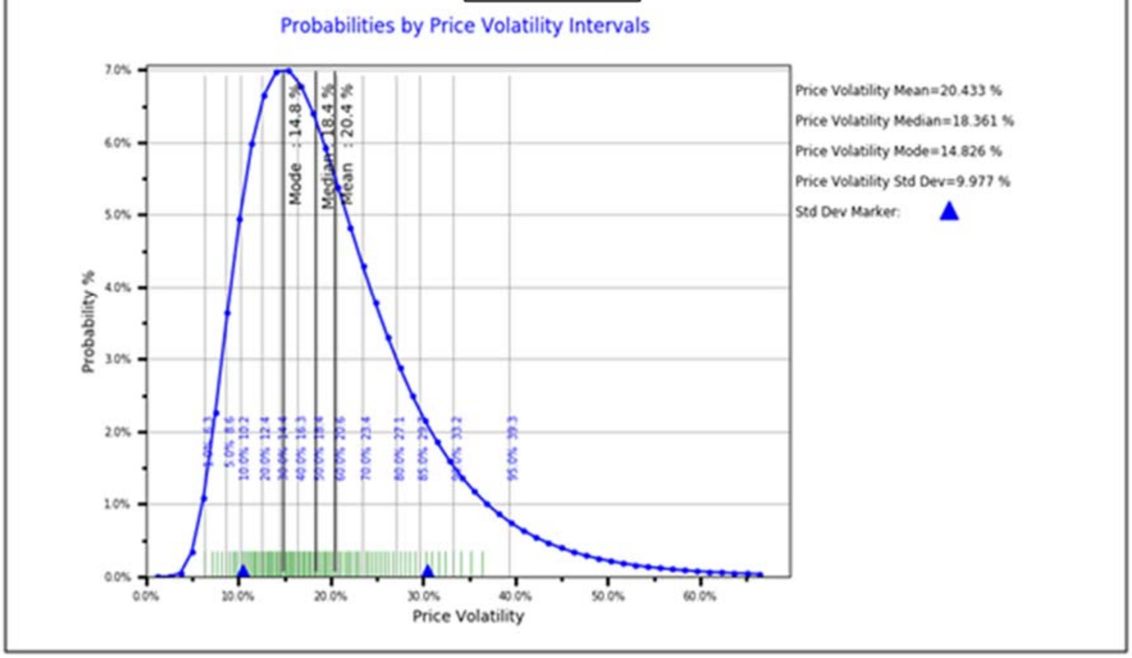
[117] The use of guideline companies to estimate the subject company's stock price volatility is consistent with the requirements of SFAS 123(R) at paragraphs 23 and A22.

[118] Subject to possible adjustment described in SFAS 123(R), using the historical volatility of stock over the most recent time period corresponding in length to the expected period of restriction is consistent with the requirements of the pronouncement. *See* paragraph A21 of the SFAS.

[119] Proper probability analysis should consider the extent to which serial autocorrelation is present in the price data. That is, current volatility levels may tend to predict the next period's price volatility. It may be possible to counteract the effects of serial autocorrelation by extending the time period of price volatility investigation.

[120] The VIX distributes log-normally because the values are never less than zero.

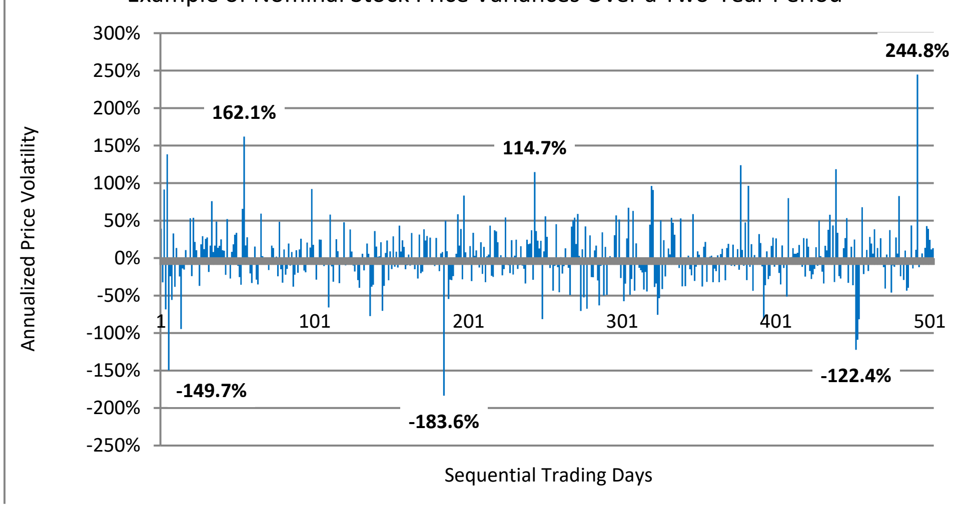
Figure 6.16



Section 3.A – Measuring Price Volatility

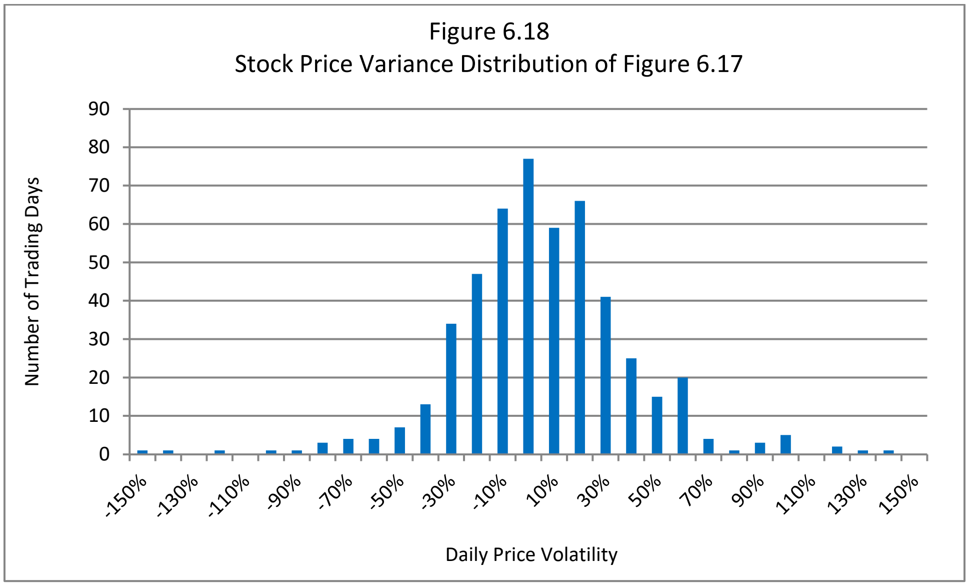
Price volatility is a statistical measure of the dispersion of returns. If the daily results are stated in nominal terms, then they will exhibit variation similar to the example presented in Figure 6.17. The average price variance of this example is just 1.7%, because of the net effects of positive and negative daily price fluctuations.

Figure 6.17
Example of Nominal Stock Price Variances Over a Two-Year Period

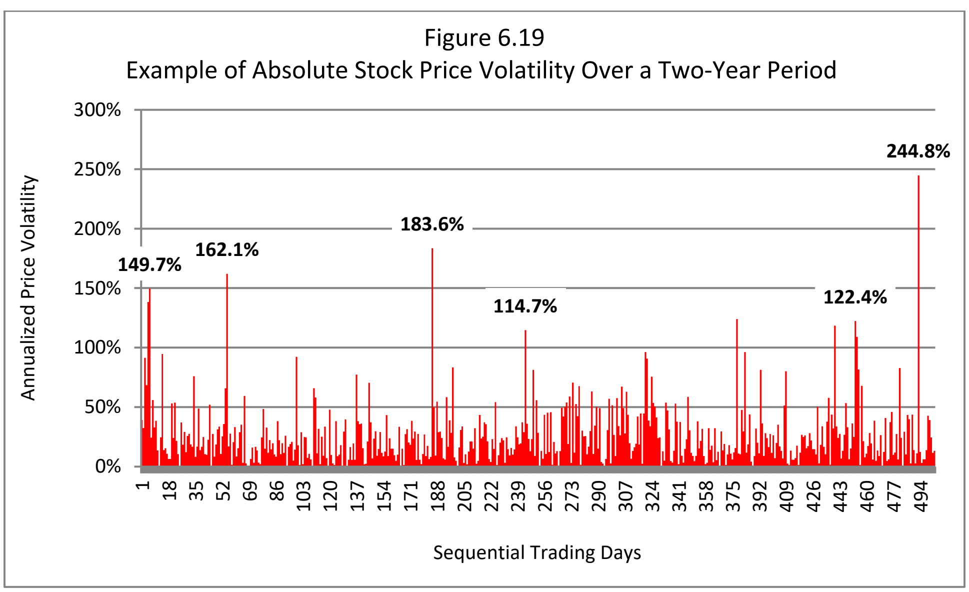


Source: VFC DLOM Calculator, www.dlomcalculator.com

Figure 6.18 presents the distribution of the Figure 6.17 daily price variances. Not surprisingly, the frequency of daily price variance yields a "normal" distribution—the familiar bell-shape that in this example centers on the 1.7% average and has a median value of -0.6%.

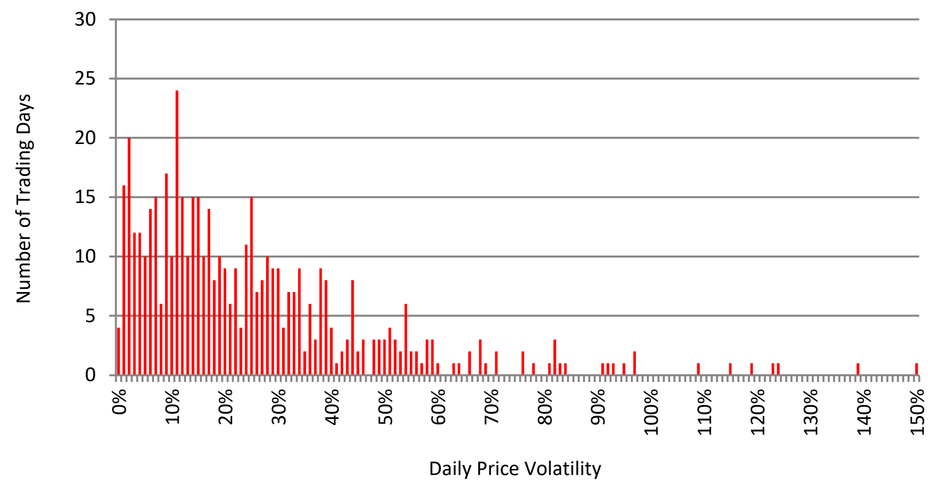


Price volatility is stated in absolute numbers, however, meaning that negative variances are converted to positive numbers. On that basis, a negative 1% price change and a positive 1% price change represent two 1% price changes. Figure 6.19 shows the effect of converting the Figure 6.17 price volatilities to absolute numbers. All price variances are now positive.



Stating price volatility in absolute numbers also changes the distribution of the data. Because no values are negative, the distribution becomes "log-normal" as Figure 6.20 shows. The average price volatility of the example stated in absolute terms is 25.7% and the median price volatility is 18.6%. The most frequently occurring price volatility shown by Figure 6.20 is approximately 11%.[121] Log-normal distribution allows analysts to plot a prediction of the probability distribution of price volatility without the offsets of negative price change events.

Figure 6.20
Stock Price Volatility Distribution of Figure 6.19



Section 3.B – The Disparate Price Volatilities of Restricted Stock Issuers

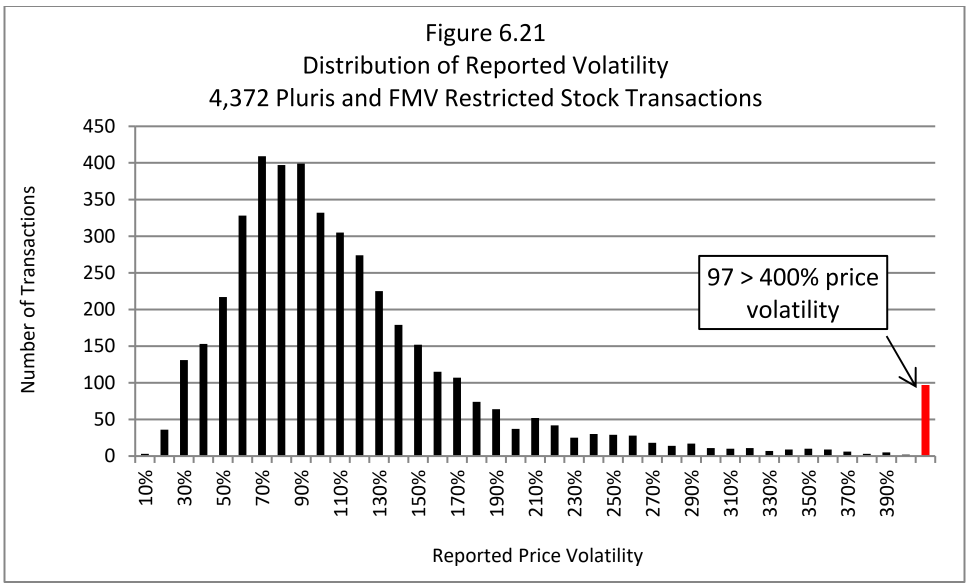
There are 4,401 restricted stock transactions in the combined Pluris® and Stout Restricted Stock Study™ ("Stout Study" or "Stout") databases. Both databases report price volatility figures as of the dates of their respective restricted stock transactions. The Stout Study offers (a) measures of market volatility based on the 1, 3, and 12-month VIX, and (b) the issuing firm's stock price volatility. The *Companion Guide* issued by FMV Opinions, the previous owner of the Stout Study, states that issuing firm stock price volatility is calculated using 12-month daily volatility expressed as a percentage.[122] No volatility is reported for 29 transactions in the Stout Study. The Pluris® database offers measures of market volatility based on (x) the VIX and (y) the issuing firm's stock price volatility based on daily and weekly prices over 3, 6, and 12 months. Therefore, Pluris® offers six different company-specific volatility measures. But not all Pluris®

[121] This graph is intentionally limited for presentation purposes to 150% price volatility. It therefore omits the three most extreme volatility events shown by Figures 6.17 and 6.19.

[122] *FMV Companion Guide* at page 29.

transactions report all of these values; consequently, the Pluris® database column called "volatility" is a mix of 3,338 twelve-month, 176 six-month, and 106 three-month volatilities measured using daily price changes, and 7 twelve-month, 3 six-month, and 2 three-month transactions measured using weekly price changes.

Twenty-nine of the 4,401 combined transactions of the Pluris® and Stout databases reports have no reported price volatility, which reduces the number of those with "volatility" reported to 4,372. The average price volatility reported for these 4,372 stock issuers with is 118.2%. But the range of volatilities is broad, reflecting the fact that the price volatilities of different businesses varies widely. Indeed, 97 of the restricted stock issuers are reported to have had stock price volatility in excess of 400%. As a result, the population of transactions exhibits a high standard deviation of 113.9%. Figure 6.21 is a histogram of the reported price volatilities.[123]



The mix of companies contributes to the distribution of stock price volatilities shown in Figure 6.21, but much of the variability is also likely due to timing as indicated by Figure 6.15. The database transaction closing dates are from 1980 through 2014, and span a broad range of market circumstances, so the price volatility measurement periods make a significant difference to the outcomes. And other factors also affect stock price volatility. For example, Table 6.6 shows the variation of average volatility by 1-digit SIC code for the 4,372 restricted stock issuers presented in Figure 6.21. Timing and industry are therefore critical aspects of properly calculated price volatility assumptions.

[123] The 97 transactions with reported price volatility greater than 400% are aggregated for presentation purposes.

Table 6.6
Average Price Volatility by SIC Code of
4,372 Restricted Stock Issuers

From	To	Number of Transactions	Average Volatility
0	1000	11	120.7%
1000	2000	600	126.7%
2000	3000	799	113.7%
3000	4000	938	116.9%
4000	5000	237	111.4%
5000	6000	226	118.9%
6000	7000	415	79.2%
7000	8000	781	139.5%
8000	9000	339	114.0%
9000	10000	26	204.9%

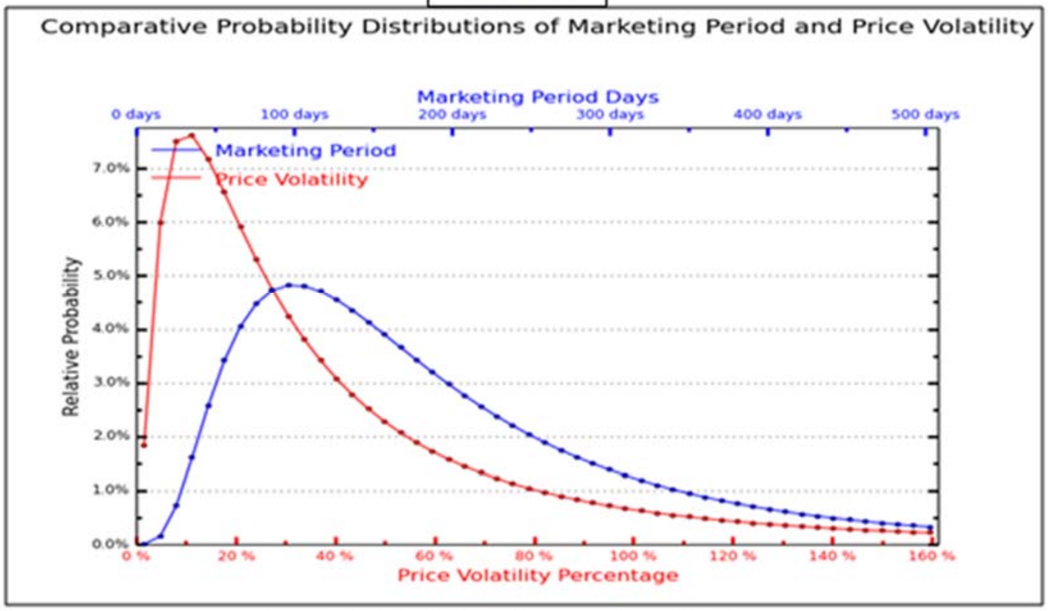
Section 4 – Enhanced Probability Estimation

Enhanced estimates of the price risk over periods of illiquidity can be crafted by determining probabilities of occurrence associated with marketing periods and price volatilities using historical information and forward looking analytical techniques.[124] For example, Figure 6.16 is a probability analysis of historical price volatility based on the mean and standard deviation of the Figure 6.15 data. Such log-normally estimated distributions provide important asymmetrical informational lacking in the traditional application of option formula models, and provide the ability to account for the full range of likely outcomes faced by an investor who holds an illiquid asset.

Figure 6.22 demonstrates example distributions for a marketing period estimate and a price volatility estimate. This graph allows the user to visualize how the time and price probabilities may differ. Importantly, each point along the distribution curves has a determinable probability of occurrence. Different datasets, different analyses, and professional judgment will of course yield different characteristics and considerations that result in different statistical means and distributions than those shown in Figure 6.22.

[124] An example of a forward-looking technique is the GARCH method for predicting near-term price volatility. A discussion of GARCH is beyond the scope of this document.

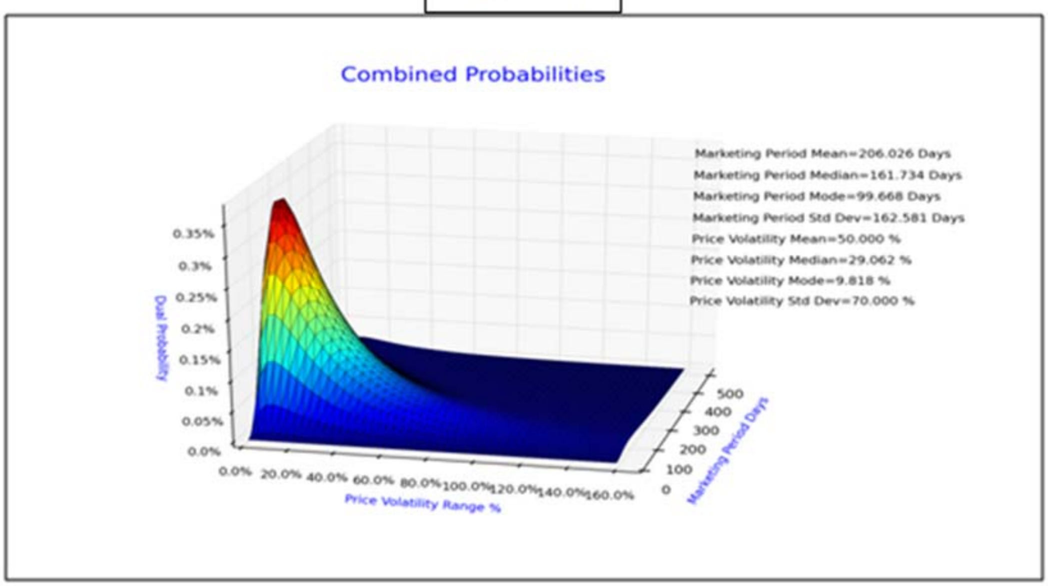
Figure 6.22



Source: VFC DLOM Calculator, www.dlomcalculator.com

The probability distributions for time and price volatility can be combined so that the probability of occurrence of each combination can be calculated. Figure 6.23 presents this three-dimensionally. Probability-based DLOMs can then be calculated on each point of combination.

Figure 6.23



Source: VFC DLOM Calculator, www.dlomcalculator.com

Section 5 – The DLOM Effects of Restricted Stock versus Private Company Illiquidity Periods

The DLOMs required for valuing privately held businesses are greater than for publicly held businesses because the private company marketing periods are much longer than the public company marketing periods. For example, Table 6.1 reported that the average time period for 10,381 sales listed in the *BIZCOMPS®* database was 214 days and that the standard deviation was 176 days. And for example, Table 6.3 reported that the average processing period for 5,157 approved SEC Form S-1 filings is 97 days and that the standard deviation was 105 days. The different time periods allow consideration of private company versus restricted stock DLOMs. We start with the time assumptions below and use them to calculate double probability VFC Longstaff and VFC Black-Scholes DLOMs:[125]

- SEC approval for new S-1 filing:
  - Average time period is 97 days.
  - Standard deviation is 105 days.
- Controlling interest private company marketing period:
  - Average time period is 214 days.
  - Standard deviation is 176 days.
- Price volatility as per Figure 6.23:
  - Average is 50%.
  - Standard deviation is 70%.

Table 6.7 reports the double probability DLOMs that result from the above assumptions. The VFC Longstaff private company DLOM is 58.7% — about 37% greater than the 42.9% VFC Longstaff restricted stock DLOM. The VFC Black-Scholes private company DLOM is 29.0% — about 46% greater than the 19.9% VFC Black-Scholes restricted stock DLOM. For each formula the entirety of the DLOM difference is attributable to the different time period risks of private companies versus the restricted stocks of publicly traded companies.

Table 6.7
Private Company DLOMs Are Larger than Restricted Stock DLOMs.
Their Illiquidity Periods Are Longer

	Double Probability DLOMs	
	VFC Longstaff	VFC Black-Scholes
Restricted stock DLOM	42.9%	19.9%
Private company DLOM controlling interest	58.7%	29.0%

Source: VFC DLOM Calculator using the Double Probability Function

[125] The VFC Longstaff and VFC Black-Scholes formulas are discussed in Chapter 7.

How should DLOMs be estimated for non-controlling interests in privately held companies? Some practitioners argue that there is no empirical evidence to support the marketing periods of non-controlling interests because the transactions reported in the best available databases—*BIZCOMPS* and *DealStats*—represent controlling interests. But there is at least one potential buyer for any non-controlling interest, and that is the controlling interest. Assuming control versus non-control to be the only marketing limitation, it seems appropriate to at least initially base non-controlling interest DLOMs on controlling interest DLOMs.

Chapter 7

LONGSTAFF FORMULA DLOMs AND THE IRS

The principal risks that investors face when trying to sell illiquid assets are price volatility and marketing time uncertainty. These risks were acknowledged in the IRS publication *Job Aide for IRS Valuation Professionals* ("IRS Job Aid")[126]

> Given two identical business interests, a higher price will be paid by investors in the market for the business interest that can be converted to cash most rapidly, without risk of loss in value. An example is publicly-traded stock on the New York Stock Exchange, where the owner can order the sale and the proceeds are deposited in a bank account in three days.
>
> In the alternative, a lesser price is expected for the business interest that cannot be quickly sold and converted to cash. A primary concern driving this price reduction is that, over the uncertain time frame required to complete the sale, the final sale price becomes less certain and with it a decline in value is quite possible. Accordingly, a prudent buyer would want a discount for acquiring such an interest to protect against value loss in a future sale scenario.

This logic leads to the conclusion that if there is no price risk (i.e., the price is locked in with no additional price concessions or transaction costs), then there should be no DLOM.[127] And if there is no time risk (i.e., the business is can liquidated instantly without risk of loss of value), then there likewise should be no DLOM. It is when there is both a price risk and a time risk that a DLOM is necessary.

Option pricing models provide a way to directly measure the effects of price and time on securities values. They are an alternative to benchmarking DLOM with restricted stock and pre-IPO transactions and other forms of DLOM estimation. Using an option formula to estimate DLOM makes sense because such formulas incorporate the time uncertainty and price volatility considerations described by the IRS. The principal option pricing formulas used by practitioners to calculate DLOMs are Longstaff, Black-Scholes, and Finnerty.

- The Longstaff formula adapted existing option pricing formulas to estimate the upper bound of DLOM. The concept differs from equilibrium models that attempt to approximate the discount for lack of marketability based on how closely the optimal strategy approximates the buy-and-hold strategy.[128] The Longstaff formula relies solely

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[126] *Job Aide for IRS Valuation Professionals*, September 25, 2009, at page 4.

[127] An exception to this general rule is the cost of money associated with the time period necessary to sell the illiquid asset.

[128] Longstaff, Francis A., "How Much Can Marketability Affect Security Values?", *The Journal of Finance,* Vol. 50, No. 5 (Dec. 1995), 1767-1774, at footnote 1.

on time and price risk variables to hypothesize an investor with perfect market timing ability, and who is restricted from selling a security for a specific period of time.[129]

- David Chaffee III published a paper in 1993 presenting the theory that "put" option prices calculated with the Black-Scholes option pricing formula could be used to estimate DLOM.[130] But the Black-Scholes formula was designed to measure European put and call options, not DLOM. European put options represent the right, but not the obligation, to sell stock for a specified price at a specified point in time. European call options represent the right, but not the obligation, to buy stock for a specified price at a specified point in time. DLOM is not the equivalent of either. Instead, DLOM represents the risk of being unable to sell at any price for a specified period of time.

  In addition to time and price volatility variables, the Black-Scholes formula calls for stock price, strike price, risk-free rate, and dividend yield variables. Assuming zero for risk-free rate and dividend yield, the Black-Scholes formula yields lower values than the Longstaff formula for the same time and price volatility assumptions.
- The Finnerty formula is based on "Asian" options. The exercise price in Asian options is equal to the arithmetic average stock price over the option term.[131] In addition to time and price volatility variables, the Finnerty formula calls for risk-free rate and dividend yield variables. Assuming zero for risk-free rate and dividend yield, the Finnerty formula yields lower values than the Black-Scholes formula for the same time and price volatility assumptions.

"At the money" put options have also been suggested as a means of estimating DLOM. Such options represent the right, but not the obligation, to sell stock at the current price at a specified future point in time. Such options do not measure the risk of illiquidity, because the investor is not denied the opportunity to sell for a price that is higher than the put price.

UCLA professor Francis A. Longstaff's 1995 article published in *The Journal of Finance*[132] presented a simple analytical upper bound on the value of marketability using an option pricing theory designed to "look back" at the highest price that could have been realized during a period of marketing restriction. Dr. Longstaff's analysis demonstrated that discounts for lack of marketability ("DLOM") can be large even when the illiquidity period is very short. Importantly, the results of Dr. Longstaff's formula provide insight into the relationship of DLOM

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[129] Longstaff, Francis A., "How Much Can Marketability Affect Security Values?", The Journal of Finance, Vol. 50, No. 5 (Dec. 1995), 1768.

[130] *Job Aide for IRS Valuation Professionals*, September 25, 2009, at page 37.

[131] Duffy, Robert E., "Why Finnerty's Put Option Model Is the DLOM Model of Choice," *Financial Valuation and Litigation Expert*, Issue 32, August/September 2011.

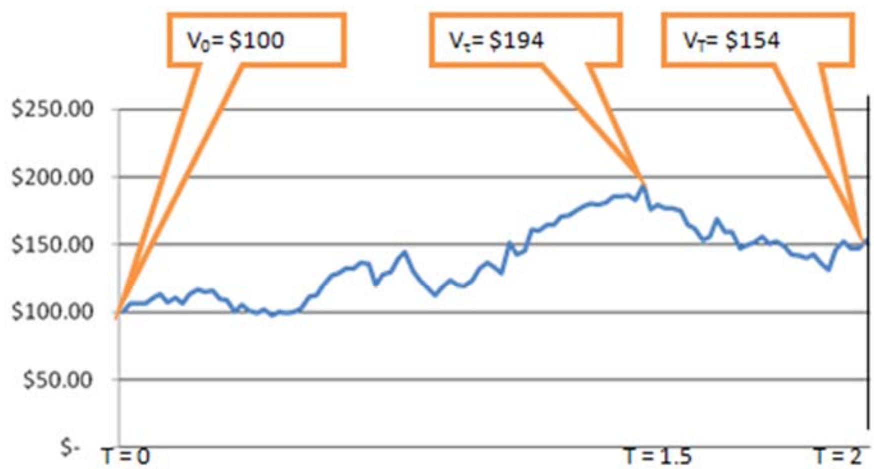
[132] Longstaff, Francis A., "How Much Can Marketability Affect Security Values?", *The Journal of Finance,* Vol. 50, No. 5 (Dec. 1995), 1767-1774.

and the length of time of a marketability restriction. Dr. Longstaff described the "intuition" behind the results of his formula as follows:

> [Consider] a hypothetical investor with perfect market timing ability who is restricted from selling a security for *T* periods. If the marketability restriction were to be relaxed, the investor could then sell when the price of the security reached its maximum. Thus, if the marketability restriction were relaxed, the incremental cash flow to the investor would essentially be the same as if he swapped the time-*T* value of the security for the maximum price attained by the security. The present value of this lookback or liquidity swap represents the value of marketability for this hypothetical investor, and provides an upper bound for any actual investor with imperfect market timing ability.

Figure 7.1 is a graphic presentation of Longstaff's description, in which an investor receives a share of stock worth $100 at time zero, but which he cannot sell for *T* = 2 years when the stock is worth $154 (present value at *T* = 0 discounted at a risk free rate of 5% = $139). If at its peak value the stock were worth $194 (present value at *T* = 0 discounted at a risk free rate of 5% = $180), then the present value cost of the restriction to the investor at *T* = 0 would be $41, or 41% of his $100 investment.

Figure 7.1
The Longstaff Formula Concept



For this sample path:

- Assume a discount rate of 5%
- With restriction, present value of T = 2 at T = 0 is 154*exp(-2*.05) = $139
- Without restriction, could have 194*exp(-1.5*.05) = $180 present value
- Cost of restriction is the difference in present values = $180 - $139 = $41

The mathematical formula of the Longstaff scenario is –

$$Discount = V\left(2+\frac{\sigma^2 T}{2}\right)N\left(\frac{\sqrt{\sigma^2 T}}{2}\right)+V\sqrt{\frac{\sigma^2 T}{2\pi}}\exp\left(-\frac{\sigma^2 T}{8}\right)-V$$

Where:

$V = current\ value\ of\ the\ investment$

$\sigma = volatility$

$T = marketability\ restriction\ period$

$N = standard\ normal\ cumulative\ distribution\ function$

Table 7.1 presents the results of the Longstaff formula at various combinations of volatility and length of time of restrictions on marketability. Figure 7.2 presents the results graphically.

Table 7.1
Longstaff Formula DLOMS at Various
Combinations of Price Volatility and Time

Restriction Period	Price Volatility 10%	20%	30%
1 Day	0.4%	0.8%	1.3%
30 Days	2.3%	4.7%	7.0%
180 Days	5.7%	11.7%	18.0%
1 Year	8.2%	17.0%	26.3%
5 Years	19.1%	41.0%	65.8%

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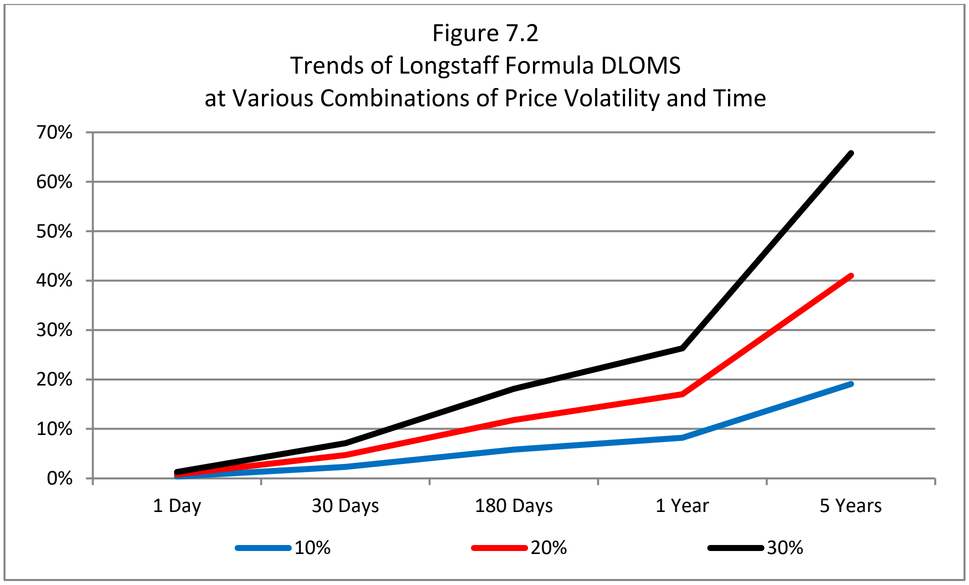

As previously stated, when DR. Longstaff presented his idea that the formula for calculating the value of a look back option with and without a liquidity restriction assumption could be used to estimate the discount for lack of marketability ("DLOM") of a financial instrument, he described his approach as quantifying the cost of illiquidity for an investor with otherwise perfect market timing ability. But Dr. Longstaff also recognized that the value of marketability, and therefore the cost of illiquidity, is less for investors with less than perfect market timing ability. Consequently, Dr. Longstaff described his approach as the "upper bound" of DLOM calculations. Consistent with the IRS Job Aid conclusion, practitioner criticisms of the Longstaff approach have focused on three perceived defects: (1) the Longstaff approach assumes perfect market timing, which no investor has; (2) Longstaff DLOMs represent "upper bound" values that are excessive; and (3) the Longstaff formula "breaks down" with variables representing long marketing periods and high price volatilities. Each of these criticisms is rebuttable as discussed below.

<u>Section 1 – The "Perfect Timing" Criticism</u>

The "perfect timing" criticism is based on a defective definition of market timing in a valuation context. The context considered by Dr. Longstaff was one of an investor with perfect market timing ability determining precisely when an investment should be sold to achieve its maximum value. Dr. Longstaff implicitly assumed that the maximum price could have been reached at any point during the investment holding period, with DLOM being the present value of the lost sale opportunity. But in a valuation context this assumption is not appropriate. Instead, the maximum price is the marketable value of the valuation subject on the valuation date. This value is the present value of the future cash benefits expected from the investment <u>before</u>

applying a DLOM. Appraisers determine this value in the ordinary course of their work, which locks the transaction timing to the valuation date.

Dr. Longstaff described the framework in which an upper bound on the value of marketability is derived as one lacking assumptions about informational asymmetries, investor preferences, and other variable that would be required for a general equilibrium model.[133] "This upper bound represents the largest discount for lack of marketability that could be sustained in a market with rational investors."[134] Dr. Longstaff recognized that the cost of illiquidity is less for an investor with imperfect market timing than it is for an investor possessing perfect market timing. "[N]onmarketability is investor-specific rather than security-specific in this framework."[135] These considerations are the basis of the "upper bound" limitation of the Longstaff methodology.

It is irrefutable that the cost of illiquidity must be less for the average investor with imperfect market timing than it is for an investor possessing perfect market timing. But the "upper bound" criticism resulting from this situation is nonetheless defective in the valuation context because it is easily resolved by using volatility estimates that represent average, not peak, volatility expectations. For example, the appraiser's volatility estimate may be based on some average or distribution of historical price volatility derived from an index or from one or more publicly traded guideline companies as discussed in Chapter 6. Using average volatility estimates in the Longstaff formula necessarily results in a value that is less than the "upper bound" value. Indeed, a value calculated using average expected volatility suggests a result that is achievable by the average imperfect investor. The resulting DLOM determined in this manner appropriately falls short of a DLOM based on perfect market timing while providing an important informational asymmetry lacking in Dr. Longstaff's more simplified framework.

As discussed in Chapter 6, enhanced estimates of DLOMs applicable to average investors can also be crafted by determining the average marketing period required to sell privately held businesses and the standard deviation of distribution around the mean. Using probability weighted marketing periods therefore provides a second important informational asymmetry lacking in Dr. Longstaff's framework. Accordingly, the "upper bound" criticism has no significance in a proper application of the Longstaff formula.

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133 Longstaff, Francis A., "How Much Can Marketability Affect Security Values?", The Journal of Finance, Vol. 50, No. 5 (Dec. 1995), 1768.

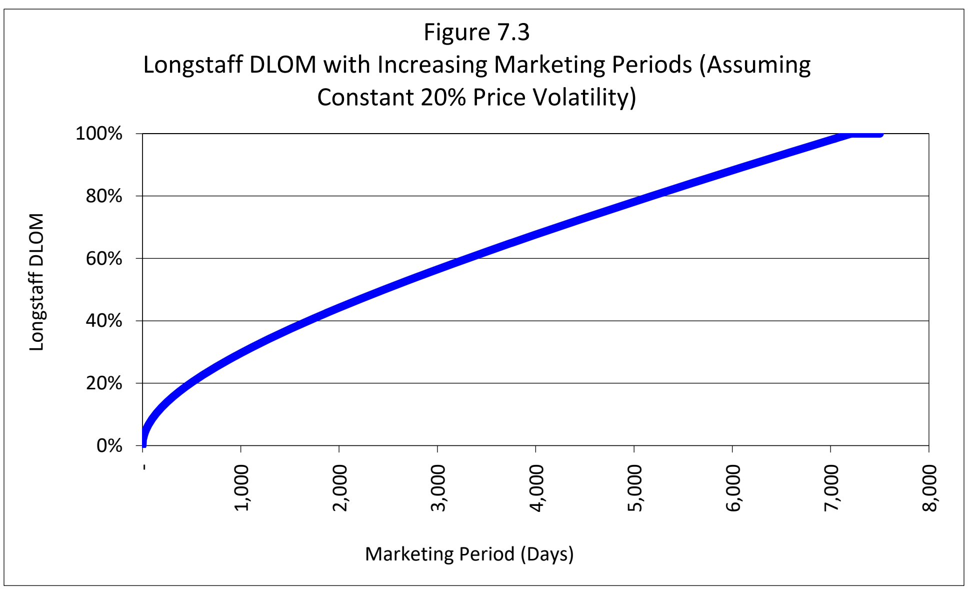
134 Longstaff, Francis A., "How Much Can Marketability Affect Security Values?", *The Journal of Finance,* Vol. 50, No. 5 (Dec. 1995), 1770.

135 Longstaff, Francis A., "How Much Can Marketability Affect Security Values?", *The Journal of Finance,* Vol. 50, No. 5 (Dec. 1995), 1769 at footnote 2.

Section 2 – The "Formula Breaks Down" Criticism

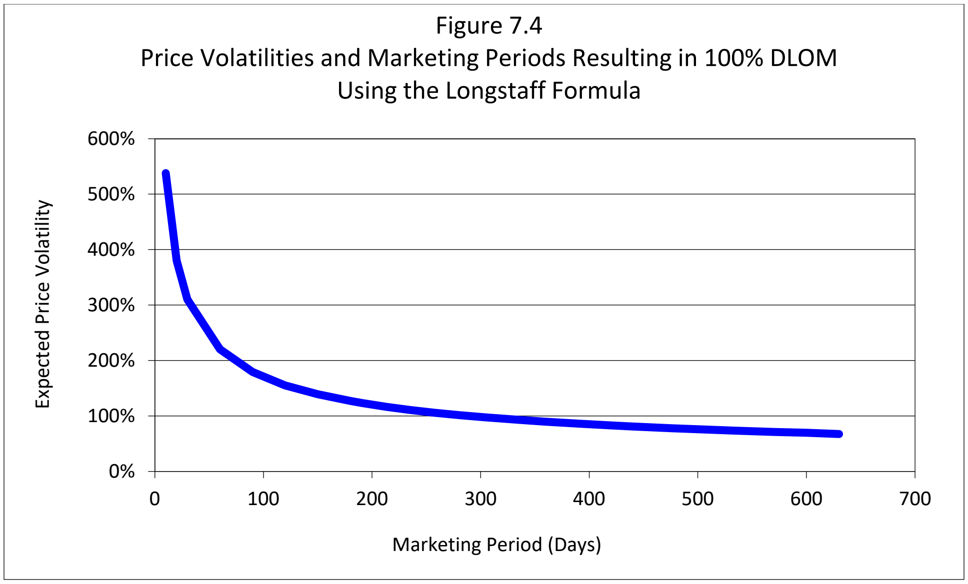
The *IRS Job Aid* makes the statement that volatilities in excess of 30% are not "realistic" for estimating DLOM using look back option pricing models. In support of this contention, the publication provides a table reporting marketability discounts in excess of 100% resulting from using combinations of variables of at least 50% volatility with a 5-year marketing period and 70% volatility with a 2-year marketing period. When that occurs, Longstaff DLOM values should simply be capped at 100%. After all, the criticism is not that the formula incorrectly calculates DLOMs below the 100% limit; merely that DLOM cannot exceed 100%.

Figure 7.3 shows the Longstaff DLOM values, capped at 100%, that result from a 20% price volatility assumption and a broad range of marketing periods. The 20% price volatility assumption approximates the historical mean of the VIX from January 2, 1990, to June 30, 2011. Note that it takes about 6,970 days – over 19 years – for the discount to reach 100% with a 20% price volatility assumption. Considering that the typical privately-held business sells in about 200 days, a criticism based on a 19-year marketing period is clearly unreasonable.[136]



As the expected price volatility increases, a shorter time is required to reach 100%. Conversely, as the expected price volatility decreases, a longer time is required to reach 100%. The graph below shows the line demarking varying combinations of sustained price volatility and marketing periods above which Longstaff DLOM values exceed 100%.

[136] The VIX peaked at 80.86% on November 20, 2008. With that assumption, the Longstaff formula requires a 450-day lockup period to reach 100% DLOM.



As previously stated, the IRS contends that volatilities in excess of 30% are "not realistic" for estimating DLOM using look-back option pricing models. In support, the *IRS Job Aid* provided a table reporting marketability discounts in excess of 100% resulting from combinations of variables of at least 50% volatility with a 5-year marketing period, and at least 70% volatility with a 2-year marketing period. The table is recreated as Table 7.2.

Table 7.2 [137]

DLOMs Summarized from IRS Job Aid

	Price Volatility		
Marketing Period	10%	40%	70%
30 Days	2.3%	9.5%	17.0%
180 Days	5.7%	24.5%	45.7%
1 Year	8.2%	36.1%	69.2%
2 Years	11.8%	53.7%	106.7%
5 Years	19.1%	93.7%	198.5%

It is obvious that if the DLOMs shown by the IRS were simply limited by practitioners to 100%, then the criticism associated with the 2-year / 70% and 5-year / 70% values shown in the above table would be at least substantially eliminated. Every instance in which the combination of time and price volatility resulted in a value greater than 100% would simply be stated as 100%.

[137] Internal Revenue Service, *Job Aid for IRS Valuation Professionals*, September 25, 2009, page 33.

Using a static time period and/or static price volatility in the Longstaff formula as the IRS did can be appropriate in situations where either or both of those variables are certain, assuming that such a situation can even exist. However, the marketing periods of assets and price risks are rarely, if ever, constants. Instead, as the discussions of these variables in other chapters showed, marketing periods and price volatility exhibit ranges of probabilistic outcomes. The solution for the appraiser is to base DLOM conclusions on a probability-based approach that accounts for the full range of predicted outcomes such as discussed in the Chapter 6.

Section 3 – The Effects of Standard Deviation on Probability Distributions

Standard deviation is a statistical measure of how dispersed data points are from the statistical mean, and reflect the probability of occurrence of a particular characteristic. Standard deviations increase as the underlying population becomes more dispersed, and vice versa. A lower standard deviation signifies that the distribution tends to be gathered closer to the statistical mean. Distributions are often depicted as "normal" (the familiar bell-shaped curve) or "lognormal" (a curve that skews more to one or the other side of the statistical mode (the characteristic with the greatest frequency of occurrence). Data that distributes normally can have negative values; In contrast, data that distributes log-normally cannot have a value less than zero. Distributions of elapsed time are always lognormal for DLOM purposes—time does not move backwards. The proper measure of price volatility is also lognormal, despite that it can be presented normally, because price volatility is the risk of price change regardless of the direction of the change.

Normal distributions with relatively low standard deviations are concentrated relatively closer to the population mean and mode, which are the same. Conversely, distributions with relatively high standard deviations exhibit are spread relatively farther from mean and mode. Lognormal distributions are different because the lowest possible value is always zero. This attribute cause the modes of high standard deviation distributions to be closer to zero than the modes of low standard deviation distributions. For example, each of the distributions shown in Figure 7.5 has a mean of 180 days, but a different standard deviation. The blue line has a standard deviation of 45 days (25% of the mean); the red line has a standard deviation of 90 days (50% of the mean); and the green line has a standard deviation of 135 days (75% of the mean). Note that in each instance the mode has moved progressively to the left of the mean, and that the mode of green line—the one with the highest standard deviation—is closest to zero while skewing the farthest to the right of the mean.

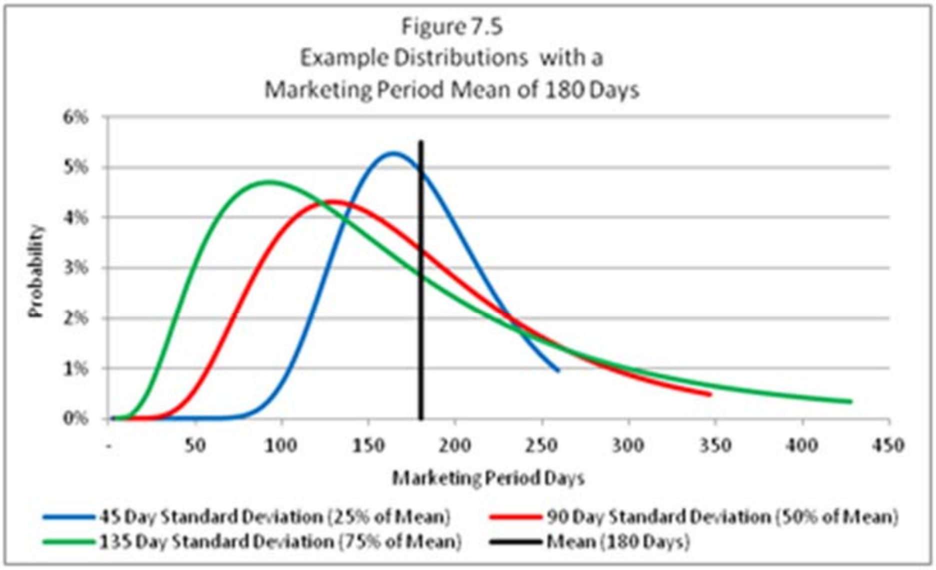


Figure 7.5
Example Distributions with a
Marketing Period Mean of 180 Days

Source: VFC DLOM Calculator, www.dlomcalculator.com

Section 4 – Adding Probability to the Longstaff Formula

Envision a population of asset sale transactions with a mean marketing period of 180 days, and a standard deviation of 180 days. Now envision that the price risk associated with the population of assets has a price volatility mean of 40% and a standard deviation of 40%. The combined probabilities would look like Figure 7.6, with the preponderance of likely outcomes concentrated around the combined modes of the distributions of the two variables. It is readily seen that the chance of greatly extended marketing periods and very high price volatilities in the envisioned scenario is remote. Calculating DLOM over the combined range of distributions would yield a value that reflects the full range of statistically predictable outcomes.

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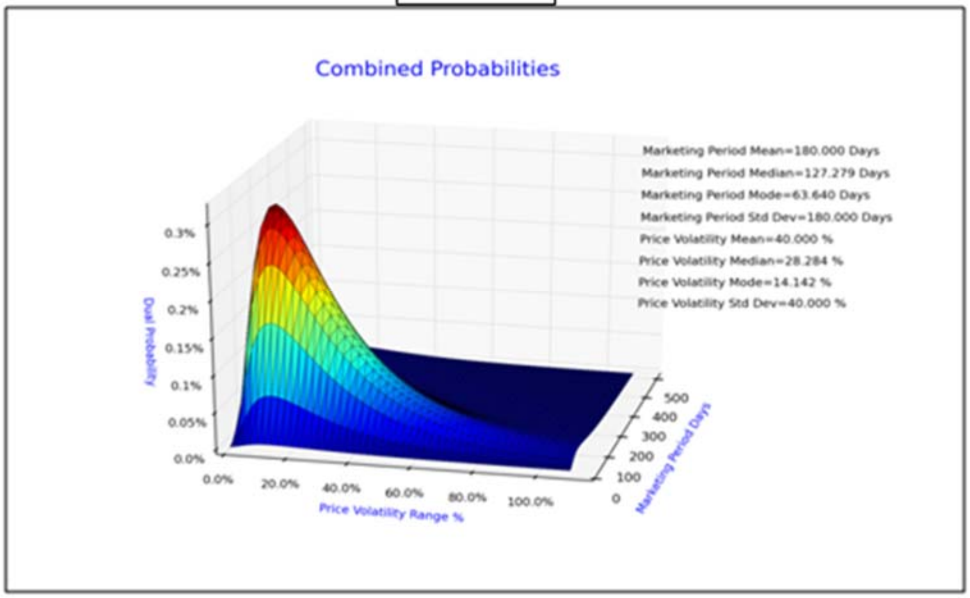
Figure 7.6



Source: VFC DLOM Calculator, www.dlomcalculator.com

Despite low probability of occurrence, the extreme combinations of marketing period and price volatility shown in Figure 7.6 (the area of dark blue) can result in points for which "raw" DLOMs exceed 100%. In this example, 68.4% of the probability combinations would result in DLOMs greater than 100% if not limited. Figure 7.7 shows where such points occur in the combined distributions, but Figure 7.8 shows that the occurrences carry little DLOM weight, contributing only about 6% to the full probability-based DLOM.

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Figure 7.7

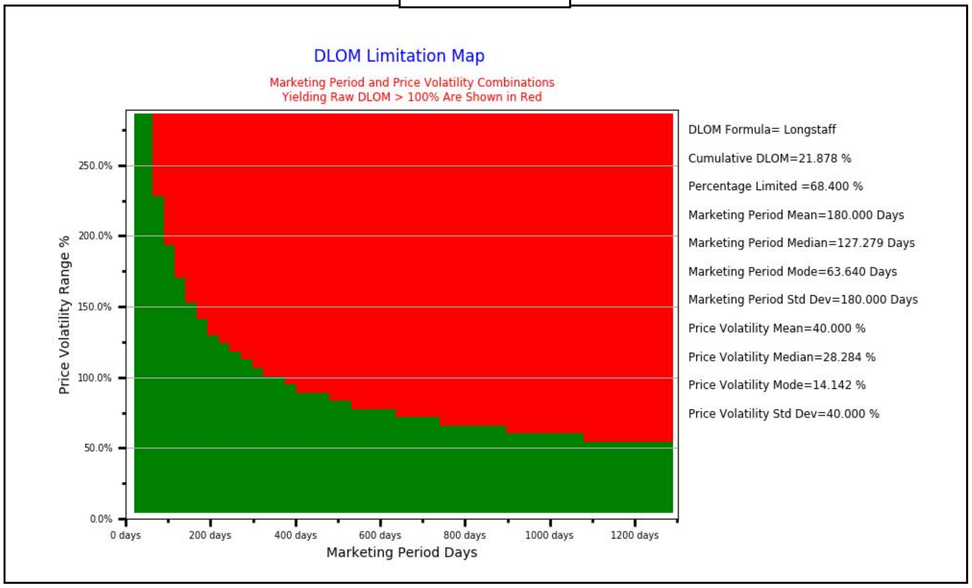
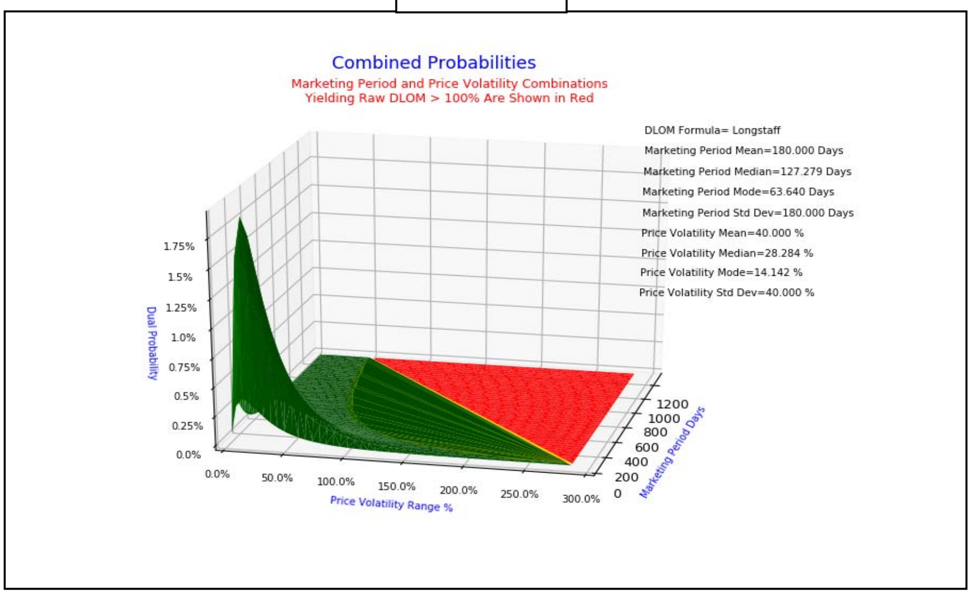
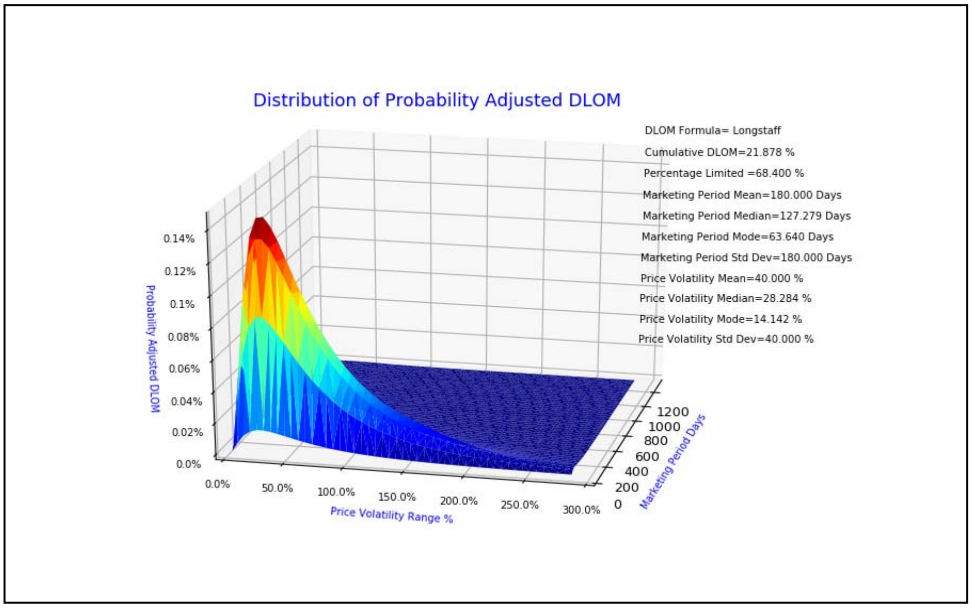


Figure 7.8



DLOMs decrease after weighting them by their probability of occurrence. In this example, the result would be a DLOM distribution as shown in Figure 7.9 that reflects a DLOM conclusion of 21.9% instead of the 24.5% raw DLOM shown in Table 7.2.

Figure 7.9



The 2.6% reduction from the Table 7.2 DLOM to the Figure 7.9 DLOM is due to applying probability to the price and time period parameters in the calculation. We know, of course, that the raw Longstaff DLOM value calculated by the IRS is less than 100%, so any reduction must be due to probability and not be due to limiting DLOM to 100%. The reduction occurs because probability shifts statistical modes closer to zero, thus proportionately reducing the number of high DLOM combinations of price volatility and time period. In this example, the statistical modes are 14.1% price volatility and 63.6 marketing period days, compared to static values of 40% price volatility and 180 marketing period days. Additionally, in this probability-based example, extreme-parameter combinations that result in DLOMs greater than the 24.5% calculated by the IRS have extremely low chances of occurring. Figure 7.8 shows this.

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<u>Section 5 – Single Probability DLOM</u>

Now let's create some probability-based alternatives to Table 7.2. First, assume that the price volatilities of Table 7.2 are static values but that the marketing periods have standard deviations equal to 50% of their means. And assume that raw DLOMs in excess of 100% are limited to 100%. The resulting single-probability DLOMs are presented in Table 7.3.

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Table 7.3 [138]
Table 1 Adjusted for Marketing Period Probability
(0.5 Coefficient of Variation)

Marketing Period Days		Probability-Based DLOM			Net Reduction from Table 7.2 Due to Probability [139]		
		Price Volatility			Price Volatility		
Mean	Std Dev	10%	40%	70%	10%	40%	70%
30	15.0	2.2%	9.2%	16.6%	0.1%	0.3%	0.4%
180	90.0	5.6%	23.8%	44.5%	0.1%	0.7%	1.2%
365	182.5	8.0%	35.2%	66.7%	0.2%	0.9%	2.5%
730	365.0	11.5%	52.3%	89.9%	0.3%	1.4%	10.1%
1,825	912.5	18.6%	84.0%	99.7%	0.5%	9.7%	0.3%

Table 7.3 reports a DLOM of 89.3% for the 730-day / 70% price volatility combination instead of the 106.7% DLOM presented by the IRS per Table 7.2. Similarly, Table 7.3 reports a DLOM of 99.7% for the 1,825-day / 70% combination instead of the 198.5% DLOM presented by the IRS per Table 7.2.

Alternatively, assume that the standard deviations of the marketing periods are equal to 200% of their means. The resulting single-probability DLOMs are presented in Table 7.4.

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[138] These probability-based DLOMs were computed using a 99.7% distribution precision. The values differ from a previously-published article that used a 95% distribution precision.

[139] The differences between the probability-based values in this table versus Table 7.2 reflect increases for price volatility and time period combinations that result in 100% DLOM calculations, and decreases for the probability of price volatility and time period combination occurrence. All net reductions below 100% DLOM from Table 7.2 are attributable to the effects of adding probability to the DLOM calculation, while all reductions to 100% are deemed attributable to imposing a 100% limitation on Longstaff-based DLOMs.

Table 7.4 [140]
Table 1 Adjusted for Marketing Period Probability
(2.0 Coefficient of Variation)

		Probability-Based DLOM			Net Reduction from Table 7.2 Due to Probability [141]		
Marketing Period Days		Price Volatility			Price Volatility		
Mean	Std Dev	10%	40%	70%	10%	40%	70%
30	60	1.9%	7.7%	13.9%	0.4%	1.8%	3.1%
180	360	4.7%	20.1%	36.1%	1.0%	4.4%	9.6%
365	730	6.7%	29.3%	50.3%	1.5%	6.8%	18.9%
730	1,460	9.6%	41.4%	65.5%	2.2%	12.3%	34.5%
1,825	3,650	15.7%	61.1%	83.3%	3.4%	32.6%	16.7%

Comparing Tables 7.3 and 7.4 reveals the effects of different probability assumptions on the different combinations of marketing period and price volatility. Contrary to intuition, larger standard deviations result in smaller DLOMs, because the statistical mode shifts closer to zero, while increased skewing of the distribution to the right of the statistical mean causes a small cumulative value of the variable.

Now let's recreate Table 7.2 assuming that the marketing periods are static but that the price volatilities have standard deviations equal to 50% of their means. And assume that raw DLOMs have been limited to 100%. The resulting single probability DLOMs are presented in Table 7.5.

Table 7.5 reports a DLOM of 80.7% for the 730-day / 70% price volatility combination instead of the 106.7% DLOM presented by the IRS per Table 7.2. Similarly, Table 7.5 reports a DLOM of 94.9% for the 1,825-day / 70% combination instead of the 198.5% DLOM presented by the IRS per Table 7.2.

[140] These probability-based DLOMs were computed using a 99.7% distribution precision. The values differ from a previously-published article that used a 95% distribution precision.

[141] The differences between the probability-based values in this table versus Table 7.2 reflect increases for price volatility and time period combinations that result in 100% DLOM calculations, and decreases for the probability of price volatility and time period combination occurrence. All net reductions below 100% DLOM from Table 7.2 are attributable to the effects of adding probability to the DLOM calculation, while all Table 7.2 reductions to 100% are deemed attributable to imposing a 100% limitation on Longstaff-based DLOMs.

Table 7.5 [142]
DLOM Adjusted for Price Volatility Probability
(0.5 Coefficient of Variation)

Marketing Period Days		Probability-Based DLOM			Net Reduction from Table 7.2 Due to Probability [143]		
		Price Volatility			Price Volatility		
Mean		10%	40%	70%	10%	40%	70%
	Std Dev	5%	20%	35%	5%	20%	35%
30		2.3%	9.5%	17.2%	0.0%	0.0%	-0.2%
180		5.7%	24.8%	45.5%	0.0%	-0.3%	0.2%
365		8.2%	36.6%	63.5%	0.0%	-0.5%	5.7%
730		11.8%	52.4%	80.7%	0.0%	1.3%	19.3%
1,825		19.3%	76.0%	94.9%	-0.2%	17.7%	5.1%

Alternatively, assume that the standard deviations of the price volatilities are equal to 200% of their means. The resulting single-probability DLOMs are presented in Table 7.6.

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142 These probability-based DLOMs were computed using a 99.7% distribution precision. The values differ from a previously-published article that used a 95% distribution precision.

143 The differences between the probability-based values in this table versus Table 7.2 reflect increases for price volatility and time period combinations that result in 100% DLOM calculations, and decreases for the occurrence probabilities of all price volatility and time period combinations. Net reductions below 100% DLOM from Table 7.2 are attributable to the effects of adding probability to the DLOM calculation, while reductions to 100% are attributable to imposing a 100% limitation on the IRS's calculations. The net increases in DLOM per this Table 7.5 are because of the tight statistical distribution that keeps the mode close to the mean, and because probability does not always fully offset the increased DLOMs (i.e., greater than the IRS value but less than 100%) from the more extreme combinations of price volatility and time period under such circumstances. For example, referring to the 365-day line in Table 7.5, the 36.6% DLOM reflects 20 unlimited price volatility occurrences greater than the 40% mean, while the 63.5% DLOM reflects just 5.

Table 7.6 [144]
DLOM Adjusted for Price Volatility Probability
(2.0 Coefficient of Variation)

		Probability-Based DLOM			Net Reduction from Table 7.2 Due to Probability [145]		
Marketing Period Days		Price Volatility			Price Volatility		
Mean		10%	40%	70%	10%	40%	70%
	Std Dev	20%	80%	140%	5%	20%	35%
30		2.2%	9.3%	15.7%	0.1%	0.2%	1.3%
180		5.6%	21.1%	32.5%	0.1%	3.4%	13.2%
365		8.1%	28.0%	41.2%	0.1%	8.1%	28.0%
730		11.4%	35.9%	50.1%	0.4%	17.8%	49.9%
1,825		17.3%	47.7%	62.0%	1.8%	46.0%	38.0%

As with the marketing period variable, comparing Tables 7.5 and 7.6 reveals that larger standard deviations of price volatility result in correspondingly smaller DLOMs. This is because of the same skewing effect associated with the higher standard deviations of Table 7.6.

Comparison of Tables 7.3 and 7.4 with Tables 7.5 and 7.6 reveals another aspect of probability-based Longstaff DLOMs. One might anticipate that toggling the input value of the marketing period and price volatility variables would result in the same DLOMs. It does not because the Longstaff formula squares price volatility but does not square time. This magnifies the effects of changes in price volatility assumptions relative to changes in time period assumptions.

Section 6 – Double Probability DLOM

Now let's consider how DLOM is affected by combining the inputs of Tables 7.3 and 7.5, and, alternatively the inputs of Tables 7.4 and 7.6. A double probability scenario involving dual 0.5 coefficients of variation is shown with Table 7.7. With this low standard deviation assumption there is a slight further diminution of the resulting DLOMs:

[144] These probability-based DLOMs were computed using a 99.7% distribution precision. The values differ from a previously-published article that used a 95% distribution precision.

[145] The differences between the probability-based values in this table versus Table 7.2 reflect increases for price volatility and time period combinations that result in 100% DLOM calculations, and decreases for the probability of price volatility and time period combination occurrence. All net reductions below 100% DLOM from Table 7.2 are attributable to the effects of adding probability to the DLOM calculation, while all reductions to 100% are deemed attributable to imposing a 100% limitation on Longstaff-based DLOMs.

Table 7.7 [146]
DLOM Adjusted for Marketing Period and Price Volatility Probabilities
(0.5 Coefficient of Variation)

Marketing Period Days		Probability-Based DLOM			Net Reduction from Table 7.2 Due to Probability [147]		
		Price Volatility			Price Volatility		
Mean		10%	40%	70%	10%	40%	70%
	Std Dev	5%	20%	35%	5%	20%	35%
30	15	2.2%	9.2%	16.7%	0.1%	0.3%	0.3%
180	90	5.5%	24.0%	43.7%	0.2%	0.5%	2.0%
365	182.5	8.0%	35.3%	60.6%	0.2%	0.8%	8.6%
730	365	11.5%	50.2%	77.1%	0.3%	3.5%	22.9%
1,825	912.5	18.8%	72.5%	92.5%	0.3%	21.2%	7.5%

Alternatively, a double probability scenario using dual 2.0 coefficients of variation is shown with Table 7.8. In contrast to Table 7.7, however, this high standard deviation scenario demonstrates a substantial diminution of the resulting DLOMs:

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[146] These probability-based DLOMs were computed using a 99.7% distribution precision. The values differ from a previously-published article that used a 95% distribution precision.

[147] The differences between the probability-based values in this table versus Table 7.2 reflect increases for price volatility and time period combinations that result in 100% DLOM calculations, and decreases for the probability of price volatility and time period combination occurrence. All net reductions below 100% DLOM from Table 7.2 are attributable to the effects of adding probability to the DLOM calculation, while all reductions to 100% are deemed attributable to imposing a 100% limitation on Longstaff-based DLOMs.

Table 7.8 [148]
DLOM Adjusted for Marketing Period and Price Volatility Probabilities
(2.0 Coefficient of Variation)

Marketing Period Days		Price Volatility			Net Reduction from Table 7.2 Due to Probability [149] Price Volatility		
Mean		10%	40%	70%	10%	40%	70%
	Std Dev	20%	80%	140%	20%	80%	140%
30	60	1.8%	7.3%	12.4%	0.5%	2.2%	4.6%
180	360	4.5%	16.6%	25.9%	1.2%	7.9%	19.8%
365	730	6.4%	22.1%	33.1%	1.8%	14.0%	36.1%
730	1,460	9.0%	28.6%	41.0%	2.8%	25.1%	59.0%
1,825	3,650	13.7%	38.6%	52.3%	5.4%	55.1%	47.7%

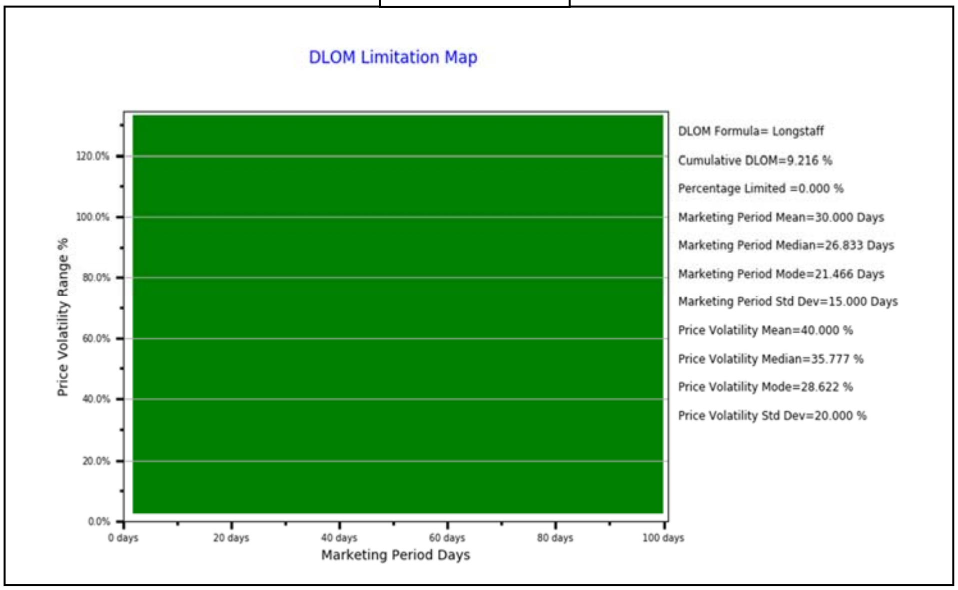
Applying probability to both the time and price volatility variables of the Longstaff formula reduces DLOM relative to corresponding single probability calculations. This result occurs because the compounding effect of two probability functions further skews the distribution of likely outcomes. The downward effect on DLOMs becomes greater as standard deviations increase. For example, referring to Table 7.7, the 30-day / 15-day and 40% volatility / 20% volatility combination has a DLOM of 9.2%, and none of the underlying calculations exceed 100% DLOM. See Figure 7.10

[Intentionally Blank]

[148] These probability-based DLOMs were computed using a 99.7% distribution precision. The values differ from a previously-published article that used a 95% distribution precision.

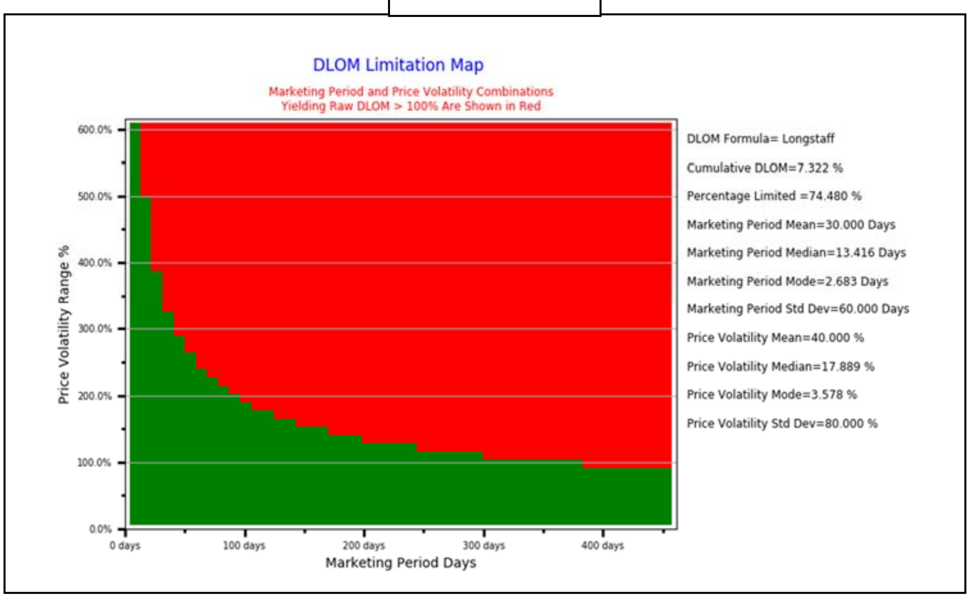
[149] The differences between the probability-based values in this table versus Table 7.2 reflect increases for price volatility and time period combinations that result in 100% DLOM calculations, and decreases for the probability of price volatility and time period combination occurrence. All net reductions below 100% DLOM from Table 7.2 are attributable to the effects of adding probability to the DLOM calculation, while all reductions to 100% are deemed attributable to imposing a 100% limitation on Longstaff-based DLOMs.

Figure 7.10



In comparison, referring to Table 7.8, the 30 / 60 and 40% 80% combination has a lower DLOM of 7.3% despite that 74.5% of the underlying calculations equal or exceed 100% DLOM. See Figure 7.11.

Figure 7.11



This occurs because the predicted marketing time periods and price volatilities skew far to the right of the mean as associated uncertainty (i.e., the standard deviation) increases. And as standard deviations increase, the probability associated with each parameter goes down. Thus, a 30-day mean and 15-day standard deviation, and a 40% price volatility mean and 20% price volatility standard deviation have the distributions shown in Figure 7.12, and the combined distribution shown in Figure 7.13. Figure 7.12 shows the most extreme prediction of time period is about 100 days and that the most extreme prediction of price volatility is about 130%. This circumstance results in the large conical concentration shown in Figure 7.13.

In contrast, the 60-day and 80% price volatility standard deviation alternatives have the distributions shown in Figure 7.14, and the combined distribution shown in Figure 7.15. Now the distribution modes have shifted significantly closer to zero while the most extreme prediction of time period is about 450 days and the most extreme prediction of price volatility is about 400%. These stretched out predictions have very low probabilities of occurrence and result in the concentration of greater-probability parameters well to the left of the statistical mean, and, in this discussion, well to the left of those in Figure 7.13. Parameters bunched closer to zero yield lower probability based DLOMs, and properly so.

Figure 7.12

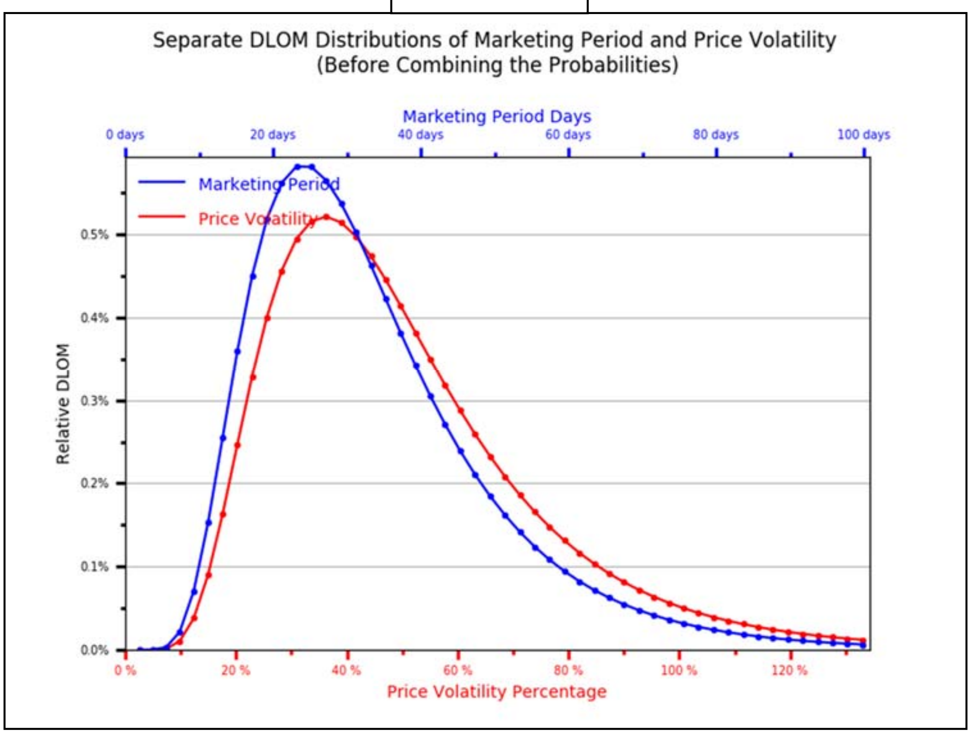


Figure 7.13

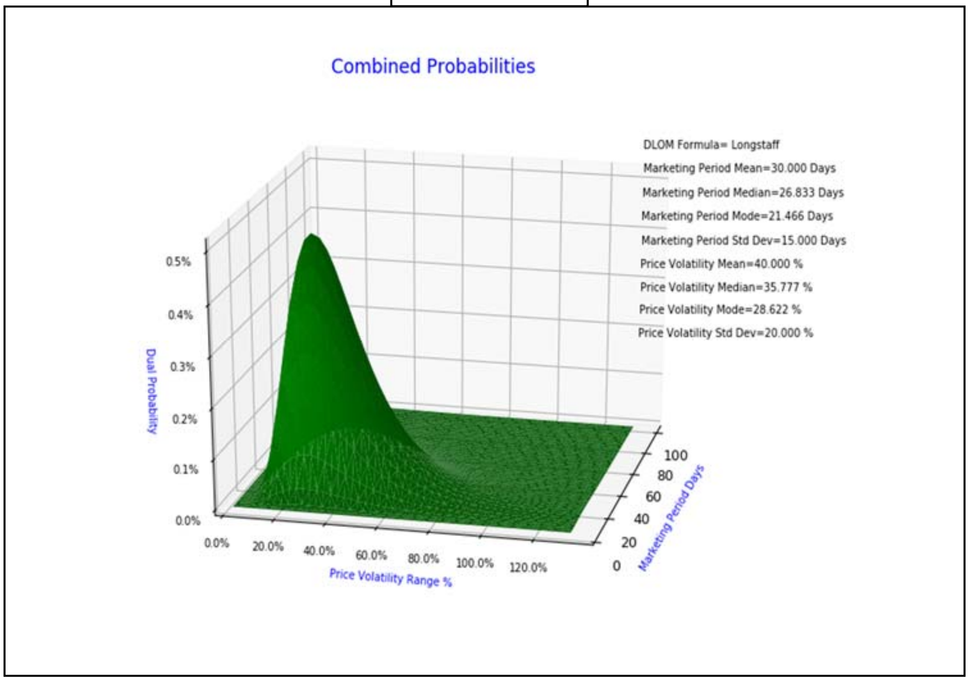


Figure 7.14

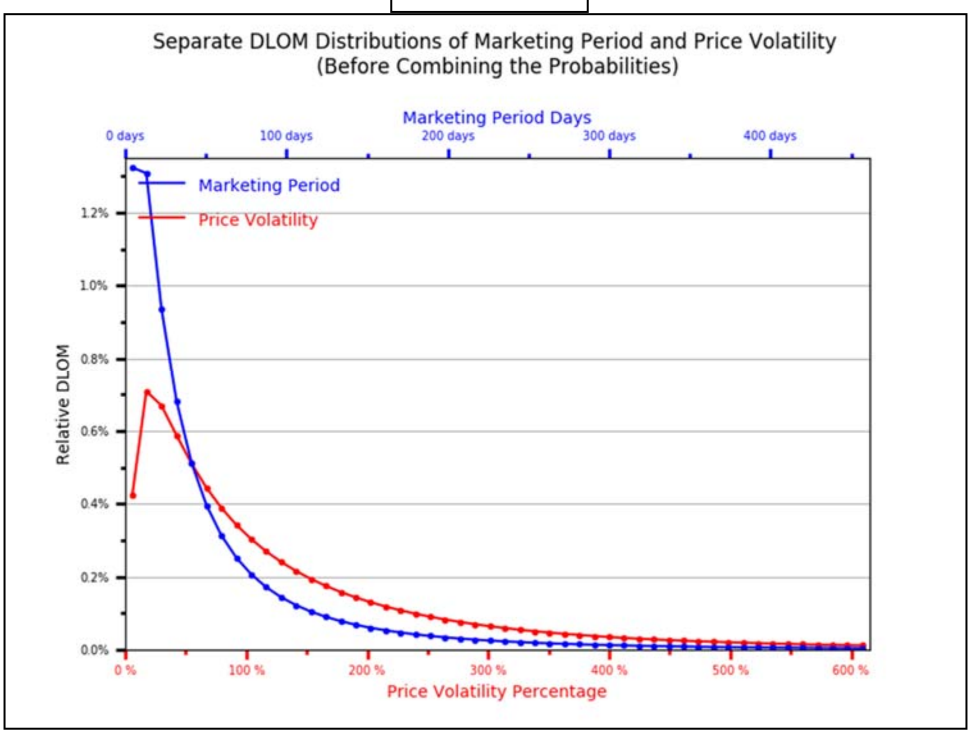
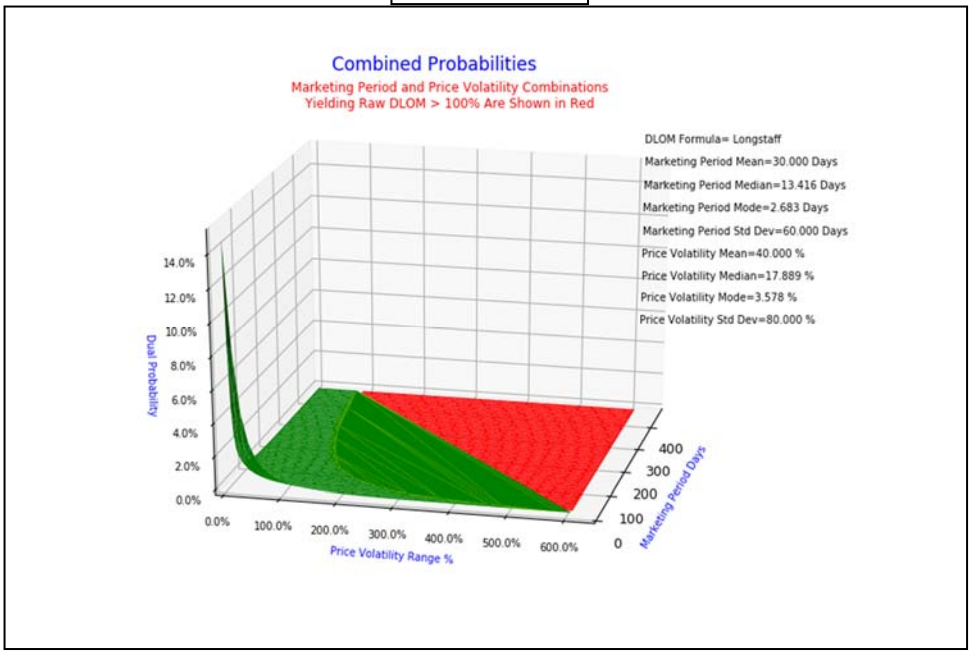


Figure 7.15



Combined Probabilities
Marketing Period and Price Volatility Combinations
Yielding Raw DLOM > 100% Are Shown in Red
DLOM Formula= Longstaff
Marketing Period Mean=30.000 Days
Marketing Period Median=13.416 Days
Marketing Period Mode=2.683 Days
Marketing Period Std Dev=60.000 Days
Price Volatility Mean=40.000 %
Price Volatility Median=17.889 %
Price Volatility Mode=3.578 %
Price Volatility Std Dev=80.000 %
Dual Probability
14.0%
12.0%
10.0%
8.0%
6.0%
4.0%
2.0%
0.0%
0.0%
100.0%
200.0%
300.0%
400.0%
500.0%
600.0%
Price Volatility Range %
400
300
200
100
0
Marketing Period Days

Chapter 8

PRICE VOLATILITY AND DISCOUNTS FOR LACK OF MARKETABILITY

Section 1 – The Reliability of Stock Price Data for Price Volatility Estimation

It became desirable in the course of this research to obtain historical price data for the Pluris® and Stout restricted stock issuers in order to independently calculate average price volatilities and related standard deviations based on split-adjusted closing prices. That effort began with historical price data available on Yahoo! Finance. It was necessary to reject Yahoo! as a source, however, because the price data was found to be unstable. For example, moments apart Yahoo! reported the price history shown below for Snap Interactive, Inc. (STVI):[150]

Time Stamp	Date	Open	High	Low	Close	Adjusted Close	Volume
16 Jun 2017 21:27:05	January 19, 2011	$2.26	$2.27	$2.15	$0.06	$2.17	130,800
16 Jun 2017 21:36:41	January 19, 2011	$79.10	$79.45	$75.25	$2.17	$75.95	3,700

This situation led to acquiring historical price data from NASDAQ. But the publicly available NASDAQ price data is limited to 10 years. As the data for the most recent day is added to the NASDAQ website, the data for the oldest day is dropped. Accordingly, price data was not readily obtainable from NASDAQ prior to about mid-2007. Additional price data was obtained from Alpha Vantage, Inc., which describes itself as a leading provider for real time and historical stock market data.[151] Alpha Vantage offers about 20 years of historical price data. Unfortunately a similar instability of data exists for Alpha Vantage as for Yahoo!. Therefore, the following algorithm was employed by the VFC DLOM Calculator® to verify price data before calculating average price volatilities and related standard deviations:

1. NASDAQ prices were used to the extent available for each issuer and restricted stock transaction closing date. The ticker symbol was required to match those of the restricted stock issuer in the Pluris® and Stout databases.
2. If the ticker symbol in NASDAQ matched that of the restricted stock issuer, and if price data was available for the issuer prior to the restricted stock transaction closing date, then the available daily closing prices up to 250 trading days reported by NASDAQ were used to calculate the issuer's average and standard deviation stock price volatility.
3. Alpha Vantage was searched to determine if the issuer's ticker symbol exists in its database. If the issuer's ticker symbol was found in the Alpha Vantage database, the

[150] http://archive.is/di1vK and http://archive.is/DEwol

[151] https://www.alphavantage.co/#about

Alpha Vantage price data was compared to the NASDAQ price data for the relevant time period to identify discrepancies. If the NASDAQ and AlphaVantage prices differed by more than 10% on all days then it was considered a failure of price verification.

The combined Pluris® and Stout databases include 4,372 transactions with reported price volatilities. This population exhibits a low 6.2% R-square of logarithmic correlation with the reported transaction discounts. The R-square of logarithmic correlation is an even lower 1.82% using the average price volatilities calculated by the VFC DLOM Calculator®. But R-square does not present the entire story of the relationship between price volatility and restricted stock discounts. The correlation of price volatilities to restricted stock discounts is greatly affected by how the discount is measured. For example, discounts reported by Pluris® for transactions with associated warrants can be considered unreliable. And the correlation of price volatilities to restricted stock discounts can be greatly affected by the discount negotiation between the restricted stock issuer and its buyer. There is likelihood that restricted stock discounts include components that are not equivalent to discounts for lack of marketability. Such components may not be responsive to stock price volatilities and sale restriction periods. Additionally, registration rights can affect the size of negotiated discounts. Consequently, a more refined correlation analysis is made by removing certain classes of transactions from the analysis of the combined databases. The following removals were made for this research project:

- Twenty-nine transactions in the Pluris® and Stout databases that have no reported price volatilities were removed. This reduced the analytical population of transactions with reported volatilities to 4,372. This condition did not affect the regression analyses based on price volatilities generated by the VFC DLOM Calculator®.
- 1,867 transactions with accompanying warrants were removed from both price volatility analyses. Removing the transactions with warrants reduced the analytical population of transactions with price volatilities reported by Pluris® and Stout to 2,505, and the analytical population of transactions with VFC DLOM Calculator® price volatilities to 2,534.
- 1,687 restricted stock transactions were removed from the VFC price volatility calculations, because the transactions are more than 10 years old and daily stock prices are not available from NASDAQ. These removals reduced the analytical population of VFC price volatility transactions to 847. The group of Pluris® and Stout volatility transactions was not affected by this elimination.
- 427 restricted stock transactions were removed because the issuers appear to no longer be publicly traded and their price histories are not available from NASDAQ. These removals reduced the analytical population of VFC price volatility transactions to 420.

- The stock price volatility of 13 restricted stock issuers was zero. These transactions were removed from the analytical population of VFC price volatility transactions, which reduced the population count to 407. The group of Pluris® and Stout volatility transactions was not affected by this elimination.
- Two Stout Study transactions duplicated transactions that Pluris® reported having warrants. These Stout transactions were removed from the analytical populations, reducing the Pluris® / Stout count to 2,503 transactions. One transaction was removed from the set of VFC price volatility transactions for the same reason, which reduced that count to 406 transactions.
- The Pluris® and Stout databases contain a number of the other duplicate restricted stock transactions over-and-above the two with warrants. Removing the Pluris® duplicate transaction in each case reduced the Pluris® / Stout analytical set by 196 transactions to a count of 2,307, and reduced the VFC DLOM Calculator® by 48 transactions to 358.
- All remaining transactions with zero or negative discounts were removed. This reduced the Pluris® / Stout analytical population by 382 to a count of 1,925 transactions, and reduced the VFC DLOM Calculator® analytical population by 91 to a count of 267 transactions.
- 67 transactions within the set of VFC price volatility transactions had price histories that failed the price verification tests. This reduced this analytical set from 267 to 200 transactions.

Table 8.1 shows the removal process described above, and shows that the process increases the R-square of correlation of transaction discounts and issuer price volatilities by a factor of 3.8 to 23.48% for the Pluris/Stout price volatility data set, and by a factor of 9.8 to 17.90% for the VFC DLOM Calculator® price volatility data set. The removal process demonstrates that transaction quality and characteristics materially affect the correlation of price volatility and restricted stock discounts. The footnotes to Table 8.1 show that the regression relationships are statistically significant each step of the removal process.

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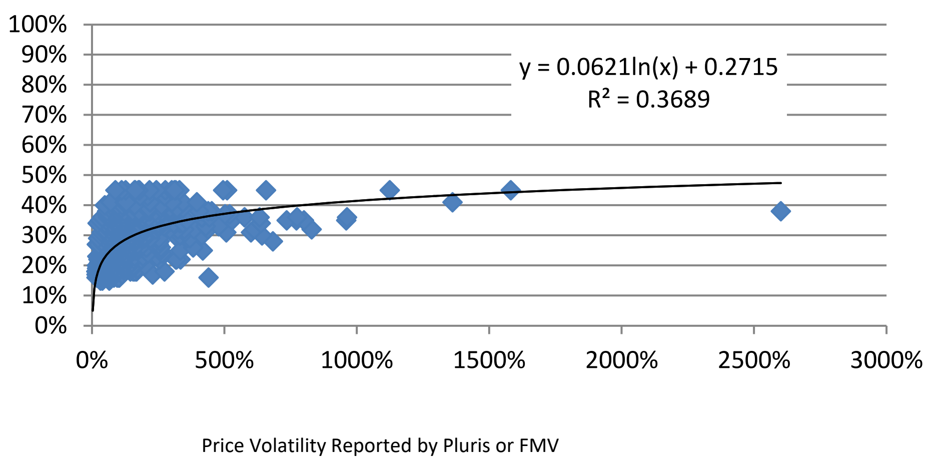
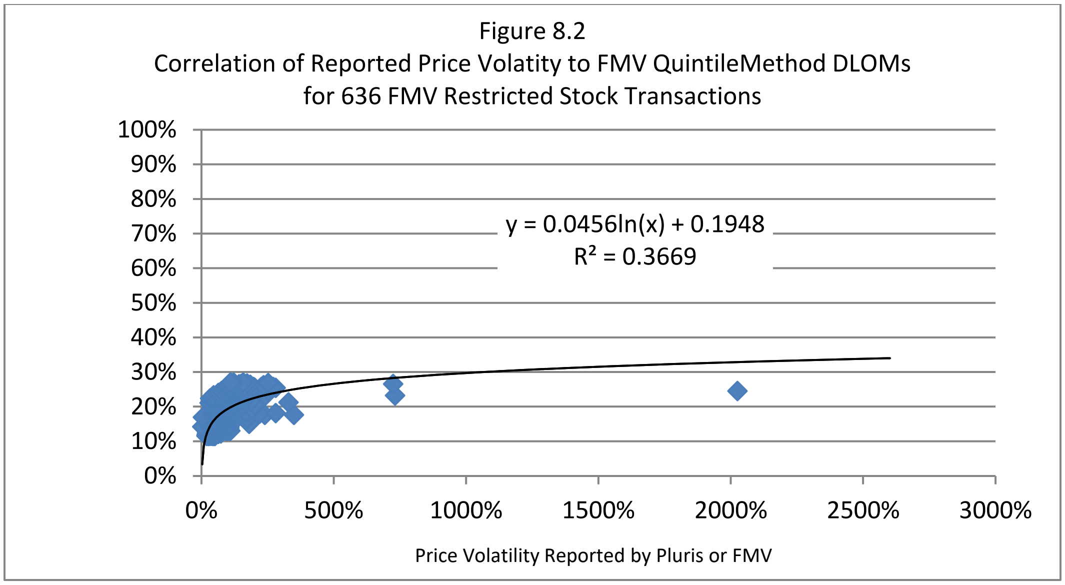
If the (a) "Stout duplicates for which Puris® has warrants," and (b) "Pluris® transactions with Stout duplicate" had not been removed, then (1) the Pluris® / Stout dataset would be comprised of 2,109 transactions with an R-square of correlation with the corresponding restricted stock discounts of 24.22%, and (2) the VFC DLOM Calculator® dataset would be comprised of 235 transactions with an R-square of correlation with the corresponding restricted stock discounts 18.48%.

Consideration of the financial significance of price volatility is further advanced by comparing the price volatilities reported by Pluris® and Stout to the resulting DLOMs of the quartile and quintile-based methods promulgated by Pluris® and Stout, respectively. Only 1,851 of the 1,925 transactions shown in Table 8.1 had all of the parameters required by the Pluris® and Stout DLOM calculation methodologies, thus limiting this next analysis. Accordingly, quartile-based DLOMs were calculated for 1,229 Pluris® transactions and quintile-based DLOMs were calculated for 622 Stout transactions.

Figure 8.1 shows the regression results for 1,230 Pluris® restricted stock transactions for which DLOMs were calculated using the Pluris® quartile-based Method 1 methodology. The R-square of logarithmic correlation is 36.89%. Figure 8.2 shows the regression results for 636 Stout restricted stock transactions for which DLOMs were calculated using the Stout quintile methodology. The R-square of logarithmic correlation is 36.69%. These results are superior to the 23.8% R-square of correlation between the reported price volatilities and the restricted stock discounts of the refined 1,925-transaction population of Pluris® and Stout restricted stock transactions.

Figure 8.1
Correlation of Reported Price Volatity to Pluris Quartile Method DLOMs
Calculated for 1,230 Pluris Restricted Stock Transactions

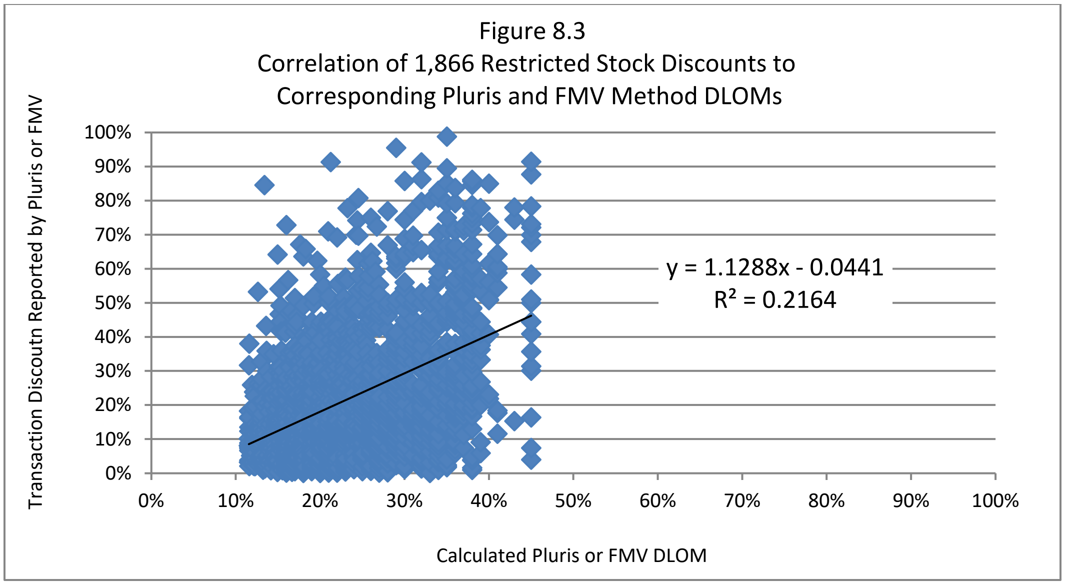




But Figures 8.1 and 8.2 also show the fundamental problem of quartile and quintile methodologies. All of the Pluris® DLOMs fall within the range of 15% to 45%, while all of the Stout DLOMs fall within the range of 11.6% to 27.0%. These results are in stark contrast to the fact the restricted stock discounts for the same transactions range from 0.1% to 98.8%.

Figure 8.3, below, results from graphing the calculated quartile and quintile DLOMs against the restricted stock discounts of the 1,866 combined transactions of Figures 8.1 and 8.2. Assuming that restricted stock discounts represents DLOM, then DLOMs should correlate linearly with the discounts. Note, however, (1) that the 21.64% R-square of correlation is significantly lower than the R-squares shown in Figures 8.1 and 8.2—the DLOMs are less correlated to the transaction discounts than to the reported volatilities; and (2) the quartile and quintile-based DLOMs are bookended between 11.6% and 45% regardless of the corresponding restricted stock discount. The vertical distribution of the reported restricted stock discounts ranges from 0.1% to 98.8%. For example, in Figure 8.3 the 35% DLOM tranche applies to myriad transactions with discounts ranging from 1.8% to 98.8%. Using the quartile or quintile methodologies requires that practitioners accept the illogical notion that disparate transactions with a very widely distributed range of discounts should all have DLOMs within a narrow range.[152]

[152] Some readers may disagree that the grouping of disparate transactions as discussed here is illogical, arguing that the point of the grouping is to capture trends in discounts associated with similar transaction characteristics thereby, among other things, removing some of the company-specific differences of the grouped firms. This argument, while convenient, is analytically flawed, because it bookends systematic results into artificially created upper and lower bounds that are contradicted empirically.



Section 2 – The Relationship of DLOMs Based on the Longstaff and Black-Scholes Formulas to Price Volatility

Probability-based DLOMs calculated using the option formulas provide strong correlations with price risk. Figure 8.4 shows the correlation of price volatility to DLOMs calculated for the 145 restricted stock transactions per Table 8.2A below using double probability VFC Longstaff and VFC Black-Scholes option formulas.[153] The two regressions show similar R-squares of correlation, but a distinct difference in the level of percentage DLOM. The average VFC Black-Scholes DLOM for the 145-transactions is 9.43%, while the average VFC Longstaff DLOM is 20.14%.

[153] Double probability DLOMs are based on combined probability distributions for price volatility and the time period of illiquidity. The VFC Longstaff methodology caps DLOM results at 100%; a cap is unnecessary for the Black-Scholes formula. The VFC Black-Scholes methodology assumes that the risk free rate and dividend yield variables are zero; the same assumption is unnecessary for the Longstaff formula. The VFC Black-Scholes line in Figure 8.5 is jagged because the trends are ordered on the Longstaff line. Spikes in the VFC Black-Scholes line are caused by the 100% DLOM limit of the VFC Longstaff formula affecting the DLOM sequence.

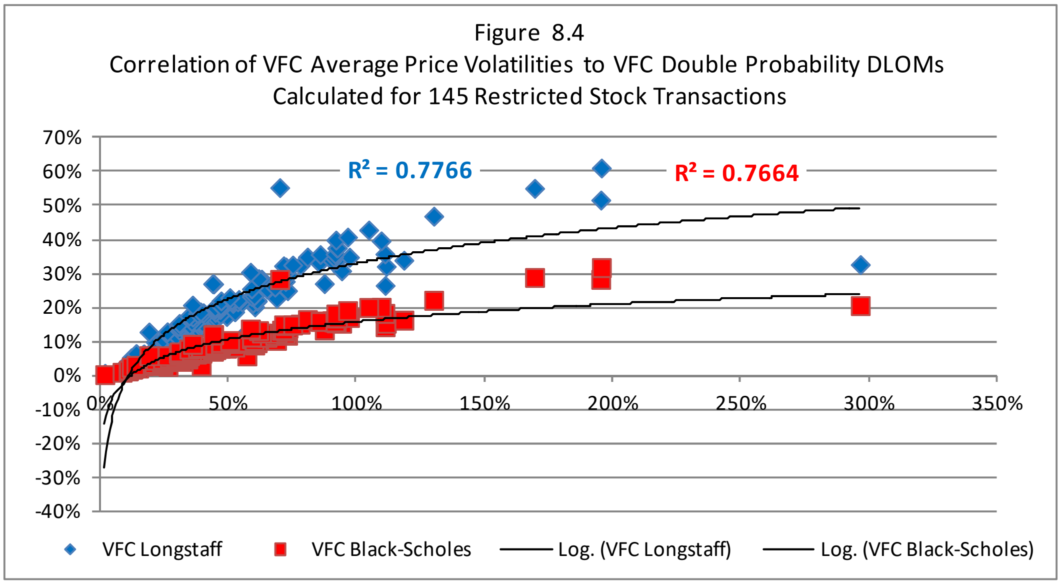
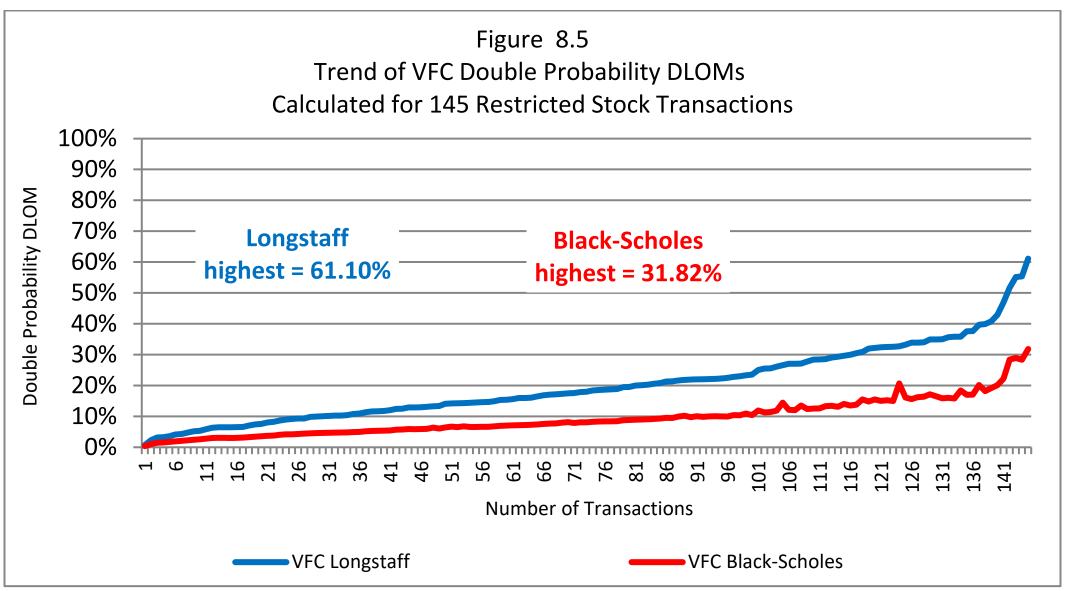
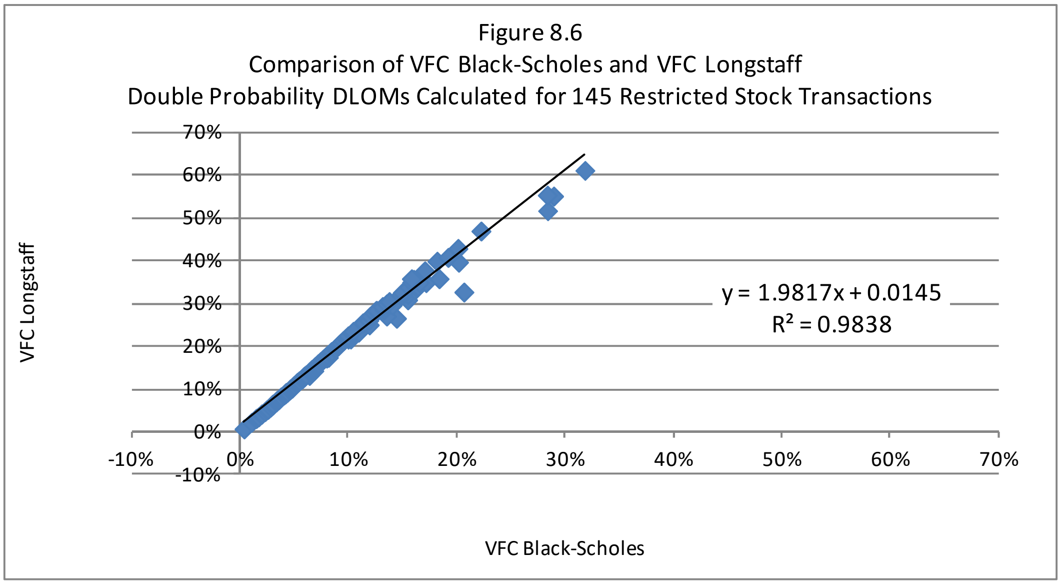


Figure 8.5 shows the trend of double probability DLOMs calculated for the 145-transaction population using the VFC Longstaff and VFC Black-Scholes methodologies. Unlike the Pluris® quartile and Stout quintile methodologies that artificially bookend DLOMs within a narrow range of values, the option formula approach allows for a full range of results commensurate with underlying assumptions of price volatility and illiquidity time periods. For this particular group of stocks, the highest VFC Longstaff DLOM is 61.1% and the highest VFC Black-Scholes DLOM is 31.8%, but with sufficiently high price volatility and/or time period assumptions DLOMs of 100% could be reached using either formula.



Double probability VFC Longstaff and double probability VFC Black-Scholes DLOMs correlate highly. Figure 8.6 shows that the VFC Longstaff and VFC Black-Scholes DLOMs for the 145-transactions set have a linear R-square of correlation of 98.38%. But, as previously stated, the average VFC Longstaff DLOM for this population of transactions is 20.14%, while the average VFC Black-Scholes DLOM for the population is 9.43%.—a difference substantially accounted for by the 1.9817 coefficient of x in the regression formula shown in Figure 8.6.



Section 3 – The Relationship of Probability-Based Option DLOMs to Restricted Stock Discounts

Figure 8.3 above shows that quartile and quintile benchmarking methods do not yield reliable DLOM estimations. But it is also obvious from Figure 8.5 that the VFC Longstaff and VFC Black-Scholes yield materially different DLOM percentages despite that Figure 8.6 shows the values correlating highly. The calculated values of both formulas cannot represent reliable DLOM estimations for business valuation—assuming that either does. We therefore now explore the extent to which the VFC Longstaff and VFC Black-Scholes formulas, combined with price volatility and illiquidity time period probabilities, are empirically supported by identifiable restricted stock transactions. Our hypothesis is that one or the other formulas should explain through linear regression analysis the majority of change in the transaction discounts by having (1) an R-square of correlation at least greater than 50%; (2) an x coefficient close to 1.0; (3) a y intercept close to zero; (4) be statistically significant at 95% probability; and (5) have a low statistical residual. The mean and standard deviation of the issuer's annualized stock price volatility over the 250 trading days prior to the transaction closing date was used to estimate the probability distribution of price volatility. The mean and standard deviation of the Securities and Exchange Commission approval time period for available time period up to ten years prior to the issuer's SIC Code was used to estimate the probability distribution of the period of illiquidity (i.e., the marketing time period).

Table 8.2A

Empirical Evidence Supports DLOMs Calculated Using the VFC Longstaff Double Probability Methodology

(Only Transactions with Positive Discounts that Passed the VFC Price Verification Test)

Number of Restricted Stock Transactions	Closing Date Range	Number of SEC Approvals in the Issuers' 4-Digit SIC Codes	Transaction Discount	Registration Rights	Linear Regressions v Transaction Discounts: VFC Longstaff DDLOM Slope	Intercept	R-Square	VFC Black-Scholes DDLOM Slope	Intercept	R-Square
Refined Restricted Stock Issuer Dataset with VFC Calculated Price Volatility Probabilities										
200 per Table 8.1	2007-2014	n/a	> 0%	DR, MR, NR, PB, No, Yes, and Unknown	DLOMs could not be calculated for 55 transactions because the issuers' reported 4-digit SIC code could not be found in the VFC database of SEC filings.					
R-Squares of Correlation and Regression Formulas Improve with More Specific SIC Codes; When Transactions with Unknown Registration Rights Are Removed; and When the Great Recession Years Are Removed										
145	2007-2014	1 or more	> 0%	DR, MR, NR, PB, No, Yes, and Unknown	0.7520	5.20%	19.93%	1.6415	4.86%	23.79%
140	2007-2014	2 or more	> 0%	DR, MR, NR, PB, No, Yes, and Unknown	0.7738	4.29%	21.23%	1.6872	3.95%	25.41%
130	2007-2014	3 or more	> 0%	DR, MR, NR, PB, No, Yes, and Unknown	0.8334	3.35%	24.28%	1.8037	3.10%	28.77%
118 per Table 6.5	2007-2014	4 or more	> 0%	DR, MR, NR, PB, No, Yes, and Unknown	0.8984	2.15%	26.94%	1.9796	1.61%	31.98%
75	2010-2014	4 or more	> 0%	DR, MR, NR, PB, No, Yes, and Unknown	1.0612	-0.19%	35.86%	2.2480	-0.29%	41.49%
59	2010-2014	4 or more	> 0%	DR, MR, NR, PB, No, Yes	**1.0109**	**-2.84%**	**54.19%**	2.0769	-2.08%	57.45%

Referring to Table 8.2A, we begin with the refined dataset of 200 transactions described in the VFC column of Table 8.1. These are the non-duplicate transactions without warrants for which the VFC DLOM Calculator® was able to obtain and cross-check daily price history to calculate the mean and standard deviation of the stock price volatility of the restricted stock issuers. That dataset shows a 17.9% R-square of logarithmic correlation of discounts to price volatility. But probability-based DLOMs could not be calculated this entire set because no SEC filings within 10 years before the transaction closing dates were found for the SIC codes of 55 issuers. As a result, probability-based DLOMs were first calculated for a population of 145 transactions. This group was specified by matching the first digit of SIC codes. This group, as

Table 8.2A shows, has a relatively low R-square of linear correlation with the transactions discounts—19.93% using the VFC Longstaff formula and 23.79% using the VFC Black-Scholes formula. The regression line slopes are also unsatisfactorily distant from 1.0—an x coefficient of 0.7520 using the VFC Longstaff formula and 1.6415 using the VFC Black-Scholes formula. Additionally, the y intercepts of the regression lines are unsatisfactorily distant from zero, with the intercepts of the VFC Longstaff and VFC Black-Scholes regression lines being 5.20% and 4.86%, respectively. Despite the relatively low R-squares of correlation for this dataset, the DLOM values are strongly statistically significant. Regressed against the transaction discounts, the VFC Longstaff DLOMs have a t-Stat of 5.966296 a P-value of 1.81978E-08, with a statistical residual of 4.5468. The VFC Black-Scholes DLOMs have a t-Stat of 6.6815057 and a P-value of 4.89244E-10, with a statistical residual of 4.3276.

A series of refinements was then undertaken to successively require more than one qualifying SEC filings within each applicable 4-digit SIC Code for estimating time probabilities. This process eventually resulted in an analytical population of 118 transactions. There are 37 Stout and 81 Pluris® transactions in this population.

Table 8.2A shows the statistical effects of refining the time period probabilities of the analytical populations. When using the VFC double probability method and the VFC Longstaff formula, there is a progressive (a) shift of the x coefficient toward 1.0; (b) shift of the y intercept toward zero; and (c) increase in R-square, as a result of more stringent time period analysis. Although the Black-Scholes formula alternative shows a progressive improvement in R-square and y intercept with more stringent time period analysis, the x coefficient unsatisfactorily moves farther from 1.0.

However, the 118-transaction population includes 43 transactions that closed during the 2007 to 2009 years of the Great Recession. Removing these reduced the analytical population to 75 transactions and further improved the VFC Longstaff regression results. The x coefficient is 1.0612 (closer to 1.0 than 0.9026 is), the y intercept is much closer to zero—just (0.19)%, and the R-square of correlation increased to 35.86%. Although the y intercept and R-square results of the VFC Black-Scholes regression likewise improve, the line slope further deteriorates to an x coefficient of 2.2480 from 1.9796.

The 75-transaction population includes 16 Pluris® transactions for which the registration rights are unknown.[154] Removing these information-deficient transactions results in a 59-transaction population that shows dramatic alignment of the VFC Longstaff DLOMs with the corresponding transaction discounts. Table 8.2A shows that this group (presented in Table 8.2B) has a 54.19% R-square of correlation; the double probability VFC Longstaff DLOMs "explain"

[154] These 16 information-deficient transactions have a low 21.92% R-square of linear correlation with transaction discounts. The regression line formula is also unsatisfactory with an x coefficient of 1.205 and a y intercept of 10.32%.

more than half of the variation in the corresponding restricted stock discounts. Also strong are the facts that the x coefficient of the regression formula for this group is virtually 1.0 at 1.0109:1 and the y intercept is acceptably close to zero at -2.84%.[155] Figure 8.7 shows the regression of the double probability VFC Longstaff DLOMs against the transaction discounts for this population.

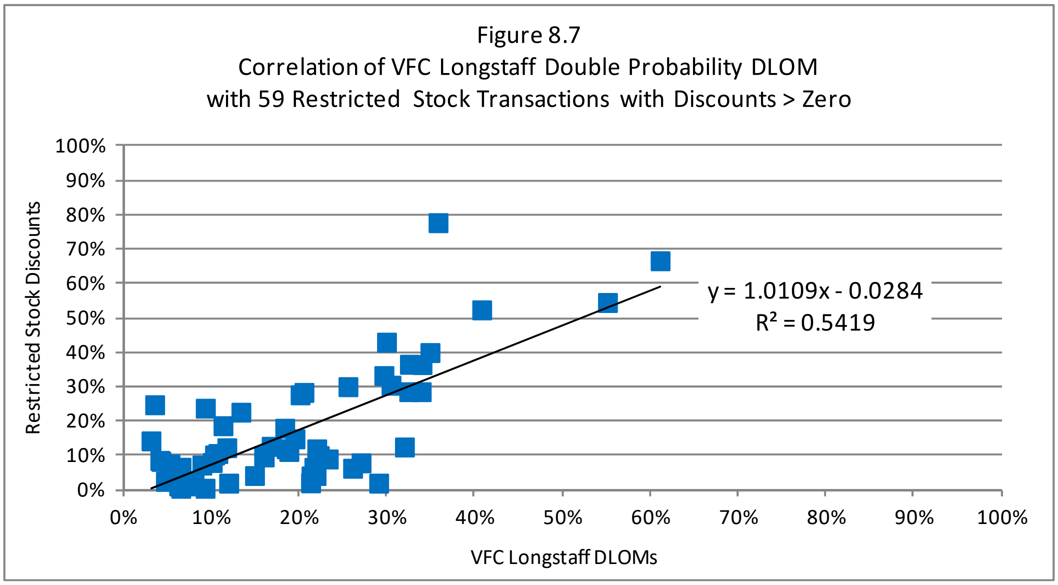


Table 8.2B
The 59-Transaction Dataset

Source	ID Number	Reported Discount	Marketing Period Mean	Marketing Period StdDev	Price Volatility Mean	Price Volatility StdDev	VFC Double Probability DLOM Longstaff	VFC Double Probability DLOM Black-Scholes	Predicted Discount	Tests for Heteroskedasticity Prediction Squared	Tests for Heteroskedasticity Predicted Residual	Tests for Heteroskedasticity Residual Squared
Stout	638	4.10%	67.8333	43.6311	59.6516%	51.7626%	21.36%	9.50%	0.187530	0.035167	-0.146530	0.021471
Stout	643	77.78%	132.8333	159.9212	111.1720%	214.6677%	35.82%	18.37%	0.333724	0.111372	0.444076	0.197203
Stout	649	30.07%	103.6500	68.1449	59.1413%	61.8621%	25.57%	11.41%	0.230045	0.052921	0.070655	0.004992
Stout	650	6.04%	110.6585	57.0890	51.6976%	81.0052%	21.95%	10.08%	0.193453	0.037424	-0.133053	0.017703
Stout	660	54.55%	74.6216	68.8859	52.5948%	50.1196%	55.11%	28.93%	0.528686	0.279509	0.016814	0.000283
Stout	667	10.00%	124.3492	100.4286	169.5737%	229.0171%	15.97%	7.21%	0.132985	0.017685	-0.032985	0.001088
Stout	672	8.33%	121.8692	129.2919	35.7527%	35.4084%	4.28%	2.01%	0.014815	0.000219	0.068485	0.004690
Stout	677	2.44%	71.8182	47.3110	12.8869%	27.1465%	8.05%	3.71%	0.052955	0.002804	-0.028555	0.000815
Stout	678	7.25%	98.0000	45.4693	21.1063%	59.4923%	8.95%	4.19%	0.062034	0.003848	0.010466	0.000110
Stout	768	9.62%	72.0204	65.5708	26.1843%	23.8520%	16.04%	7.23%	0.133701	0.017876	-0.037501	0.001406
Stout	793	11.97%	70.1250	40.1884	44.3098%	43.1219%	22.02%	9.82%	0.194148	0.037693	-0.074448	0.005542
Stout	798	6.00%	116.7500	66.9753	47.2371%	50.1905%	21.75%	10.20%	0.191462	0.036658	-0.131462	0.017282

155 The VFC Black-Scholes alternative likewise showed strong R-square improvement, increasing to 57.45%, but retained an unsatisfactory regression line.

Stout	803	12.70%	114.6047	70.3791	53.7461%	99.5948%	16.84%	7.58%	0.141779	0.020101	-0.014779	0.000218
Stout	804	40.00%	81.3214	45.5608	43.8923%	55.5479%	34.90%	17.15%	0.324397	0.105233	0.075603	0.005716
Stout	808	10.00%	123.3624	129.7032	97.3637%	156.6118%	22.30%	10.02%	0.197047	0.038827	-0.097047	0.009418
Stout	811	7.56%	119.2632	132.0038	51.3358%	54.0287%	5.33%	2.57%	0.025394	0.000645	0.050206	0.002521
Stout	814	2.26%	116.2424	80.3036	12.2318%	9.8725%	6.46%	3.04%	0.036850	0.001358	-0.014250	0.000203
Stout	815	6.54%	91.7647	87.8880	17.1355%	20.4950%	21.70%	9.97%	0.190954	0.036463	-0.125554	0.015764
Stout	819	27.71%	112.1981	139.6981	55.3802%	76.6780%	20.12%	9.04%	0.174971	0.030615	0.102129	0.010430
Stout	820	22.64%	121.4706	115.4511	44.8368%	49.2186%	13.39%	6.08%	0.106936	0.011435	0.119464	0.014272
Stout	822	17.98%	106.3288	76.0796	31.0100%	36.1273%	18.37%	8.26%	0.157311	0.024747	0.022489	0.000506
Stout	651	11.22%	129.3000	75.7787	38.0811%	47.1748%	18.84%	8.44%	0.162044	0.026258	-0.049844	0.002484
Pluris	18193	4.20%	67.8333	43.6311	61.3449%	54.6183%	21.95%	9.75%	0.193437	0.037418	-0.151437	0.022933
Pluris	19090	2.00%	112.4000	89.6298	70.8484%	93.5502%	29.06%	13.41%	0.265354	0.070413	-0.245354	0.060199
Pluris	19473	14.90%	118.4746	131.2343	44.8286%	46.2600%	19.55%	8.78%	0.169182	0.028623	-0.020182	0.000407
Pluris	20831	52.40%	124.5962	83.6677	96.6221%	113.9149%	40.80%	19.19%	0.383983	0.147443	0.140017	0.019605
Pluris	20866	12.50%	126.7059	81.3950	72.5186%	94.1763%	32.00%	14.81%	0.295031	0.087043	-0.170031	0.028910
Pluris	20891	8.60%	72.7600	65.1179	12.7068%	22.9362%	4.16%	1.97%	0.013638	0.000186	0.072362	0.005236
Pluris	20907	1.80%	61.5714	36.2091	23.3070%	22.8853%	7.58%	3.58%	0.048180	0.002321	-0.030180	0.000911
Pluris	21139	66.70%	98.0000	45.4693	195.6121%	232.7740%	61.10%	31.82%	0.589254	0.347220	0.077746	0.006045
Pluris	21173	36.50%	69.2115	63.6152	118.5695%	179.2872%	34.00%	16.39%	0.315251	0.099383	0.049749	0.002475
Pluris	21286	28.60%	116.7500	66.9753	78.1519%	109.4673%	32.49%	15.22%	0.300019	0.090012	-0.014019	0.000197
Pluris	21383	2.00%	72.6875	47.7843	33.5590%	41.6472%	12.00%	5.46%	0.092831	0.008618	-0.072831	0.005304
Pluris	21580	7.90%	198.8000	103.2577	44.0002%	43.1469%	27.07%	12.00%	0.245173	0.060110	-0.166173	0.027613
Pluris	21712	36.60%	116.7500	66.9753	75.2111%	93.5508%	32.57%	14.97%	0.300779	0.090468	0.065221	0.004254
Pluris	21764	24.80%	70.6200	64.2974	11.1556%	20.4191%	3.57%	1.70%	0.007625	0.000058	0.240375	0.057780
Pluris	21980	4.20%	119.9333	133.8942	34.8898%	46.3803%	14.93%	6.78%	0.122533	0.015014	-0.080533	0.006486
Pluris	22091	18.70%	94.0000	46.9521	28.7946%	56.8112%	11.35%	5.19%	0.086255	0.007440	0.100745	0.010150
Pluris	22337	8.00%	69.2115	63.6152	31.3135%	57.5771%	10.13%	4.64%	0.073994	0.005475	0.006006	0.000036
Pluris	22354	10.60%	125.9231	142.8094	24.4809%	30.5197%	10.77%	4.93%	0.080414	0.006466	0.025586	0.000655
Pluris	22787	9.00%	119.6730	127.0739	59.1137%	95.9189%	23.33%	10.92%	0.207372	0.043003	-0.117372	0.013776
Pluris	22847	23.80%	68.6964	63.0249	27.9384%	22.9767%	9.34%	4.37%	0.065935	0.004347	0.172065	0.029606
Pluris	22874	6.60%	68.6964	63.0249	20.7375%	43.9412%	6.58%	3.04%	0.038113	0.001453	0.027887	0.000778
Pluris	23338	12.00%	125.3000	113.5685	40.2843%	43.4961%	18.55%	8.33%	0.159087	0.025309	-0.039087	0.001528
Pluris	23494	28.60%	119.3427	132.3541	90.4812%	126.8599%	33.91%	16.22%	0.314319	0.098797	-0.028319	0.000802
Pluris	23522	1.20%	81.2500	40.1209	16.8548%	18.7369%	6.30%	2.99%	0.035227	0.001241	-0.023227	0.000539
Pluris	23574	12.30%	88.5000	56.2028	29.6919%	33.0919%	11.78%	5.39%	0.090663	0.008220	0.032337	0.001046
Pluris	23669	14.30%	77.8788	48.2781	10.9888%	66.4566%	3.16%	1.45%	0.003510	0.000012	0.139490	0.019457
Pluris	23712	0.50%	54.2000	30.0693	32.2130%	76.8820%	9.30%	4.27%	0.065621	0.004306	-0.060621	0.003675
Pluris	23911	2.50%	65.9500	61.8984	15.1290%	16.6687%	4.79%	2.29%	0.019950	0.000398	0.005050	0.000026
Pluris	24004	28.40%	109.4667	96.8737	47.0382%	44.1713%	20.56%	9.18%	0.179419	0.032191	0.104581	0.010937
Pluris	24024	43.00%	115.5492	132.0482	71.6358%	71.4412%	29.96%	13.52%	0.274481	0.075340	0.155519	0.024186
Pluris	24394	30.50%	177.1250	197.1354	58.7180%	58.7836%	30.48%	13.76%	0.279651	0.078205	0.025349	0.000643
Pluris	24480	33.30%	116.7500	66.9753	73.4052%	117.7517%	29.66%	14.05%	0.271404	0.073660	0.061596	0.003794

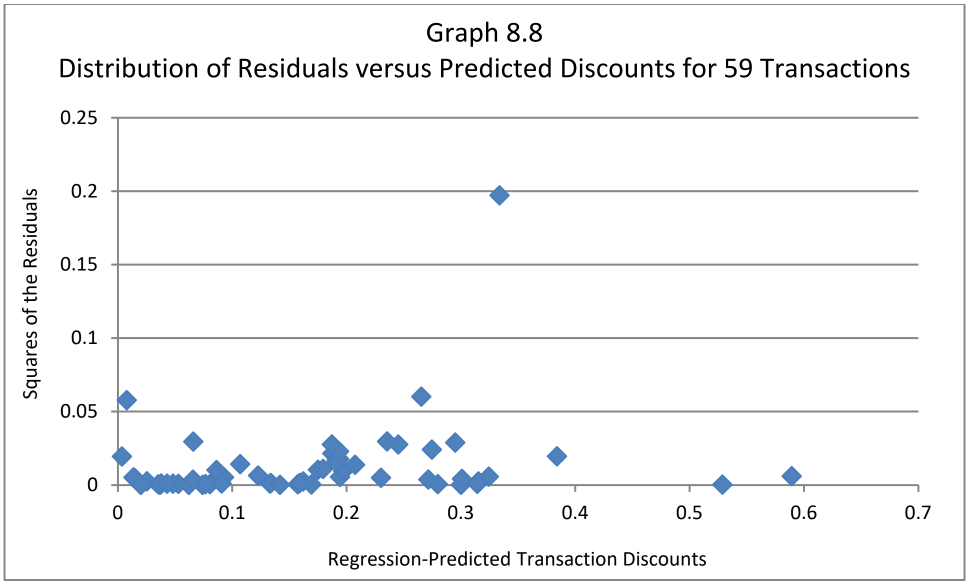
Pluris	24834	2.10%	106.5650	136.6965	51.8640%	42.6801%	21.32%	9.52%	0.187076	0.034998	-0.166076	0.027581
Pluris	24941	6.30%	95.2927	89.4575	68.2276%	83.6843%	26.09%	11.88%	0.235355	0.055392	-0.172355	0.029706
Pluris	24948	10.20%	109.6000	110.7332	24.7502%	22.2508%	10.39%	4.81%	0.076605	0.005868	0.025395	0.000645
Pluris	25039	0.50%	130.4667	87.7526	14.0652%	16.2325%	6.55%	3.09%	0.037740	0.001424	-0.032740	0.001072
Pluris	25102	1.20%	79.6389	55.5248	24.5544%	115.9724%	7.06%	3.33%	0.042917	0.001842	-0.030917	0.000956

The 59-transaction set that resulted from refining the analytical population provides strong empirical support for basing business valuation DLOMs on the VFC double probability methodology and the VFC Longstaff formula.[156] The set is statistically significant with a t-Stat of 8.21094 and a P-value of 3.07E-11. The statistical residual is 0.764.

Our next analytical process was to test the VFC Longstaff DLOMs of the 59-transaction set for heteroskedasticity, which is undesirable. The first step was to plot a scatter graph of the squares of the residuals on the y axis versus the predicted DLOMs from regression analysis on the x axis. The result is Graph 8.8, which preliminarily appears to be homoskedastic, because the variance of the y values does not appear to increase as the x values increase.

[156] Table 8.1 shows that certain apparently duplicate transactions between Pluris® and Stout were removed in arriving at the 200-transaction dataset that starts Table 8.2. If those transactions had not been removed the "refined restricted stock issuer dataset" per Table 8.1 would have 235 transactions and the 59-transaction dataset of Table 8.2 would be comprised of 73 transactions instead, with little effect on the DLOMs versus discounts regression results:

Number of Restricted Stock Transactions	Closing Date Range	Number of SEC Approvals in the Issuers' 4-Digit SIC Codes	Transaction Discount	Registration Rights	Linear Regressions v Transaction Discounts					
					VFC Longstaff DDLOM			VFC Black-Scholes DDLOM		
					Slope	Intercept	R-Square	Slope	Intercept	R-Square
59 without duplicates	2010-2014	4 or more	> 0%	DR, MR, NR, PB, No, Yes	1.0109	-2.84%	54.19%	2.0769	-2.08%	57.45%
73 with duplicates	2010-2014	4 or more	> 0%	DR, MR, NR, PB, No, Yes	0.9905	-2.95%	53.04%	2.0555	-2.35%	56.29%



The second step is to perform a Breusch-Pagan regression test of the squares of the residuals as the dependent variable and the regression-predicted transaction discounts as the independent variable. This test resulted in an F value of .121 that is not statistically significant. Step 2 does not allow us to reject the null hypothesis that the data distribute homoskedastically.

The third step was to perform an abridged White regression test of the squares of the residuals as the dependent variable against two independent variables: (1) the regression-predicted transaction values, and (2) the squares of the regression-predicted transaction values. This test result in an F value of .199 that is not statistically significant. Step 3 does not allow us to reject the null hypothesis that the data distribute homoskedastically.

We can conclude from this three-part test that the predicted values based on VFC Longstaff DLOMs distribute homoskedastically. The VFC Longstaff DLOMs of the 59-transaction set appear to be of consistent variance.

Many of the transaction discounts reported by Pluris® and Stout are very small, and may be caused by price shifts that occurred after the negotiation date of the restricted stock transactions. Referring now to Table 8.3, the further refinement of excluding transactions with discounts less than 5% had the desirable effect of moving the y intercept to virtually zero while retaining an x coefficient that is very close to 1.0 and preserving the R-square of correlation.

Table 8.3
Empirical Evidence Supports DLOMs Calculated Using the VFC Longstaff Double Probability Methodology
(Only Transactions with Positive Discounts that Passed the VFC Price Verification Test)

Number of Restricted Stock Transactions	Range of Transaction Closing Dates	Number of SEC Approvals in the Issuers' 4-Digit SIC Codes	Transaction Discount	Registration Rights	Linear Regressions v Transaction Discounts: VFC Longstaff DDLOM Slope	Intercept	R-Square	VFC Black-Scholes DDLOM Slope	Intercept	R-Square
Regression Formulas Are Further Improved if Transactions with Very Small Discounts Are Filtered Out										
59	2010-2014	4 or more	> 0%	DR, MR, NR, PB, No, Yes	1.0109	-2.84%	54.19%	2.0769	-2.08%	57.45%
45	2010-2014	4 or more	≥ 5%	DR, MR, NR, PB, No, Yes	**0.9771**	**0.28%**	**54.02%**	1.9819	1.27%	57.42%

Table 8.4 and Figure 8.8 show that the average double probability VFC Longstaff DLOM consistently approximates the corresponding average restricted stock transaction discount, providing additional empirical evidence favoring the VFC Longstaff DLOM methodology.

Table 8.4
Empirical Evidence Supports DLOMs Calculated Using the VFC Longstaff Double Probability Methodology
(Only Transactions with Positive Discounts that Passed the VFC Price Verification Test)

Number of Restricted Stock Transactions	Range of Transaction Closing Dates	Number of SEC Approvals in the Issuers' 4-Digit SIC Codes	Transaction Discount	Registration Rights	Average Restricted Stock Transaction Discount	Average Double Probability DLOM or Discount: VFC Longstaff	VFC Black-Scholes
200 from Table 8.1	2007-2014	n/a	> 0%	DR, MR, NR, PB, No, Yes, and Unknown	20.46%	n/a	n/a
145	2007-2014	1 or more	> 0%	DR, MR, NR, PB, No, Yes, and Unknown	20.34%	20.14%	9.43%
140	2007-2014	2 or more	> 0%	DR, MR, NR, PB, No, Yes, and Unknown	20.19%	20.55%	9.63%
130	2007-2014	3 or more	> 0%	DR, MR, NR, PB, No, Yes, and Unknown	20.24%	20.26%	9.50%
118 per Table 6.5	2007-2014	4 or more	> 0%	DR, MR, NR, PB, No, Yes, and Unknown	19.96%	19.83%	9.27%
75	2010-2014	4 or more	> 0%	DR, MR, NR, PB, No, Yes, and Unknown	20.64%	19.63%	9.31%
59	2010-2014	4 or more	> 0%	DR, MR, NR, PB, No, Yes	16.85%	19.48%	9.12%
45	2010-2014	4 or more	≥ 5%	DR, MR, NR, PB, No, Yes	21.40%	21.61%	10.16%

Table 8.4 shows that the average DLOMs calculated using the VFC Black-Scholes formula consistently understate the corresponding average restricted stock discounts. Other writers are of the opinion that the Black-Scholes option formula under-prices DLOM. According to Espen Robak and Lance S. Hall:[157]

> The problem with [the put option] method is that the standard option pricing methodologies available provide no insight into the value of liquidity. Indeed, one of the assumptions behind the Black-Scholes model, the most widely used valuation model for options, is that the security can be continuously traded. When valuing a put option on a security with limited marketability, the most appropriate method is either to discount the underlying security for lack of liquidity (and then apply the Black-Scholes model with the adjusted input data), or to apply a marketability discount directly to the option value indication from the Black-Scholes formula. In fact, institutions active in the "market" for private warrants purchase them at significant discounts to their calculated Black-Scholes values because of their illiquidity.[[158]]

As stated in Chapter 7, the Black-Scholes formula was created to price European puts and calls, which involve selling or buying at specific prices at specific times. Neither of these options resembles DLOM, which instead is the *inability* to buy or sell at specific prices during unspecified time periods.[159] In contrast, the Longstaff formula was specifically created to price DLOM. Referring again to Table 8.4, we see that the VFC Black-Scholes R-squares of correlation for the 59 and 45-transaction groups are slightly higher than the VFC Longstaff R-squares. The deficiency of the Black-Scholes results is that the line slope is essentially 2.0 instead of the desired 1.0. This means that DLOMs calculated using the Black-Scholes formula may be understated by as much as 100%. See Figure 8.9.

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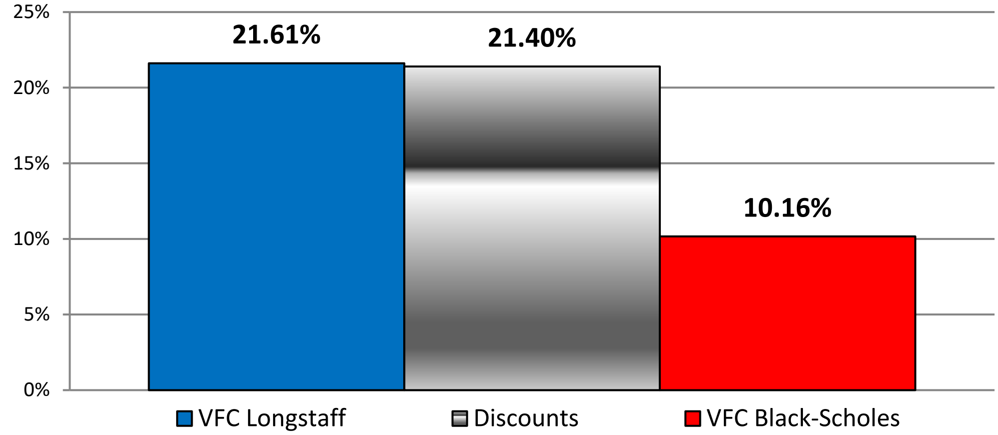
[157] Espen Robak, CFA, is the president and founder of Pluris Valuation Advisors, LLC. Lance S. Hall is the Managing Director and co-founder of FMV Opinions, Inc.

[158] Robak, Espen and Hall, Lance (2001) "Bringing Sanity to Marketability Discounts: A New Data Source," Valuation Strategies, July/August 2001.

[159] Another formula, the Finnerty formula, was created to price Asian options, involve selling at average prices at specific times. Accordingly, Asian options likewise do not resemble DLOM. Moreover, under identical assumptions the Finnerty formula yields values lower than the Black-Scholes formula.

Figure 8.9
Average Discount Compared to Average VFC Double Probability DLOM
for 45 Restricted Stock Transactions



Chapter 9

A VFC DOUBLE PROBABILITY DLOM CASE STUDY

We now take a closer look at some individual transactions reported in the Pluris® and Stout databases to further confirm the appropriateness of estimating DLOM using the VFC double probability methodology. Twelve transactions were selected from those for which VFC Longstaff DLOMs were calculated. A mix of both financial and industrial enterprises was selected, because DLOM should be predicable across all industries. General and financial information about each transaction was obtained from its source database and summarized in Tables 9.1 and 9.2. The VFC Longstaff formula is used for this case study.

Table 9.1
Industry and Transaction Data from the Pluris® and Stout Databases

Source	Restricted Stock Issuer	Ticker Symbol	SIC Code	Industry	Closing Date	Gross Placement or Proceeds
Stout	Western Alliance Bancorporation	WAL	6022	State Commercial Banks	6/27/2008	$30,156,064
Pluris	Profile Technologies, Inc.	PRTK	7389	Business Services	8/15/2008	$2,295,404
Stout	Texas Capital BancShares Inc.	TCBI	6022	State Commercial Banks	9/8/2008	$58,000,000
Stout	Opko Health, Inc.	OPK	2834	Pharmaceutical Preparations	6/2/2009	$31,000,000
Pluris	Finotec Group, Inc.	FTGI	6211	Security Brokers, Dealers, and Flotation Companies	7/31/2009	$2,000,000
Stout	Occulogix Inc.	TEAR	3841	Surgical and Medical Instruments	1/8/2010	$1,743,989
Stout	Colony Bankcorp, Inc.	CBAN	6022	State Commercial Banks	3/30/2010	$5,000,000
Stout	Boston Private Financial Holdings, Inc.	BPFH	6022	State Commercial Banks	6/22/2010	$6,267,850
Pluris	United Community Financial Corp.	UCFC	6036	Savings Institutions, Not Federal	3/22/2013	$18,079,248
Pluris	Codorus Valley Bancorp, Inc.	CVLY	6035	Savings Institutions, Federal	3/26/2014	$13,000,000

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Table 9.2
Issuer Financial Data from the Pluris® and Stout Databases

Ticker Symbol	Total Assets	Revenues (LTM)	EBITDA (LTM)	Net Profit Margin	Total Equity	Market Capitalization	Market to Book Ratio
WAL	$5,197,303,000	$336,701,000	None Stated	7.60%	$493,960,000	$528,178,000	1.07
PRTK	Zero Stated	$1,000,000	$(2,000,000)	n/a	$(1,000,000)	$32,290,000	n/a
TCBI	$4,663,236,000	$299,365,000	None Stated	9.10%	$314,917,000	$443,321,000	1.41
OPK	$19,146,000	$8,917,000	$(36,211,000)	-425.60%	$(7,777,000)	$264,947,000	-34.07
FTGI	$7,000,000	$5,000,000	$(3,000,000)	n/a	$2,000,000	$11,690,000	5.60
TEAR	$9,733,000	$869,000	$(4,860,000)	-504.60%	$6,757,000	$11,840,000	13.375
CBAN	$1,307,089,000	$75,392,000	None Stated	-25.50%	$61,918,000	$32,527,000	0.53
BPFH	$6,034,392,000	$380,335,000	None Stated	4.30%	$453,054,000	$458,363,000	1.01
UCFC	$1,821,000,000	$1,650,000,000	$(16,000,000)	-20%	$171,000,000	$118,920,000	0.70
CVLY	$1,153,000,000	$1,045,000,000	$17,000,000	20%	$108,000,000	$105,755,100	0.98

Table 9.3 presents double probability VFC Longstaff DLOMs and the underlying means and standard deviations of price volatility and illiquidity periods that VFC measured from the issuers' stock price histories and SEC Form S-1 processing times. The S-1 processing period is relevant for the prospective holder of the security because it reflects the time needed to convert the restricted stock into a stock that is publicly tradable. This is particularly relevant for restricted stocks that have piggyback rights, or mandatory registration rights. The VFC price volatility means and standard deviations were measured using the available NASDAQ price data for up to 250 trading days preceding the closing date. The VFC SEC processing time means and standard deviations were measured for a minimum of four S-1 filings in the issuer's 4-digit SIC code for up to 10 years preceding the closing date.

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Table 9.3

Double Probability VFC Longstaff DLOMs Calculated by the VFC DLOM Calculator

Ticker Symbol	Closing Date	Reported Volatility	Double Probability VFC Longstaff DLOM	SEC Processing Time for the Issuer's SIC Code (10 Year Look-Back) Mean	Standard Deviation	Issuer's Stock Price Volatility (250 Trading Day Look-Back) Mean	Standard Deviation
WAL	6/27/2008	64.20%	17.98%	76.4 days	39.5 days	46.97%	42.82%
PRTK	8/15/2008	166.00%	14.22%	98.0 days	76.1 days	39.83%	97.48%
TCBI	9/8/2008	46.10%	12.41%	73.4 days	39.2 days	33.81%	30.38%
OPK	6/2/2009	128.30%	37.63%	104.6 days	92.4 days	92.63%	88.82%
FTGI	7/31/2009	493.00%	58.93%	87.1 days	64.4 days	254.97%	403.81%
TEAR	1/8/2010	126.10%	31.60%	110.1 days	57.5 days	75.43%	99.36%
CBAN	3/30/2010	88.80%	21.97%	72.2 days	51.6 days	61.24%	65.97%
BPFH	6/22/2010	74.50%	18.84%	74.6 days	68.9 days	52.59%	50.12%
UCFC	3/22/2013	67.00%	27.03%	200.0 days	65.2 days	44.38%	53.22%
CVLY	3/26/2014	26.00%	7.62%	122.5 days	138.2 days	17.71%	17.89%

Comparing Tables 9.1, 9.2, and 9.3 provides a basis for understanding why these companies experienced dramatically different stock price volatilities in the 12 months before the restricted stock transaction dates. The different DLOM results for the 12 companies are rationalized by understanding their characteristics that cause different price risk expectations.

Why did CVLY have 250-trading day historical price volatility average of just 17.71% on March 26, 2014? We can surmise that it was because it is a financial institution with substantial assets, substantial EBITDA, a strong profit margin, and a market value to book value ratio that approximates its book assets. Investors evidently agreed that CVLY was a lower-risk stock, which is reflected in its low price volatility. Figure 9.1 shows CVLY's marketing period and price volatility risk profiles based on the means and standard deviations in Table 9.3.

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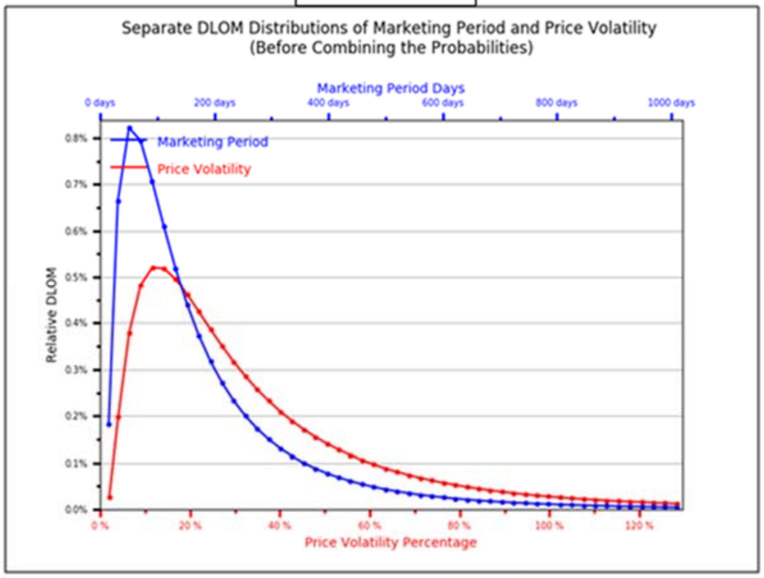
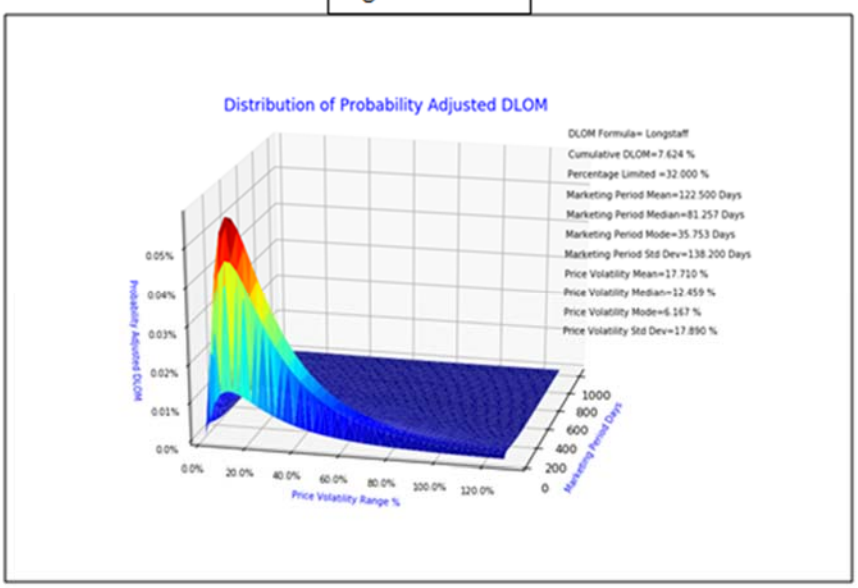
Figure 9.1 CVLY



Source: VFC DLOM Calculator, www.dlomcalculator.com

Figure 9.2 shows CVLY's double probability risk profile based on the combined risk profiles of Figure 9.1. The peaked area represents the most likely combined occurrences, and accounts for the lion's share of CVLY's 7.62% double probability VFC Longstaff DLOM shown in Table 9.3.

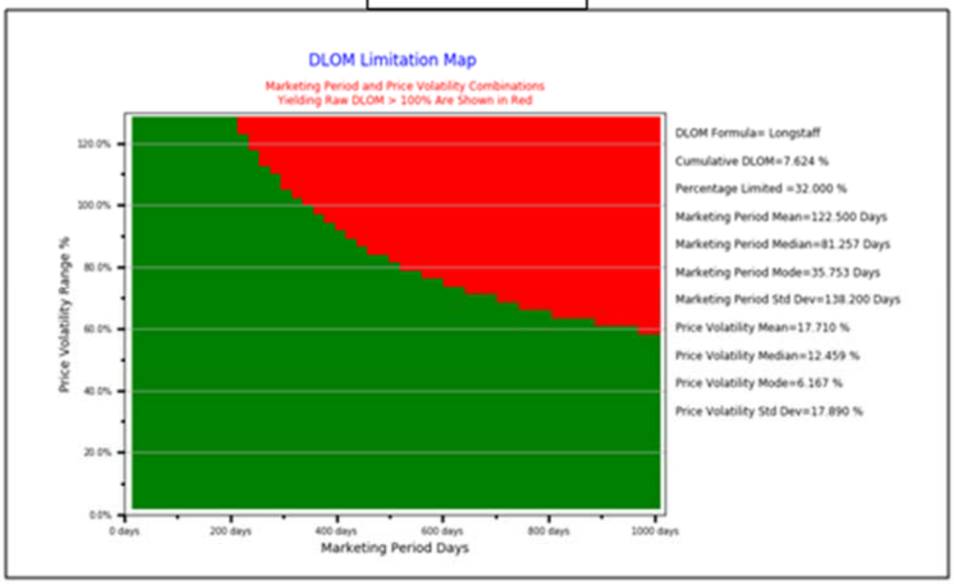
Figure 9.2 CVLY



Source: VFC DLOM Calculator, www.dlomcalculator.com

Figure 9.3 is a two-dimensional display that shows in red the proportion of CVLY's combined marketing period and price volatility probabilities that result in raw VFC Longstaff DLOMs of 100%.

Figure 9.3 CVLY



Source: VFC DLOM Calculator, www.dlomcalculator.com

Figure 9.4 is a three dimensional display that shows in red the low probabilities associated with CVLY's combinations of marketing period and price volatility probabilities that result in VFC Longstaff DLOMs of 100%.

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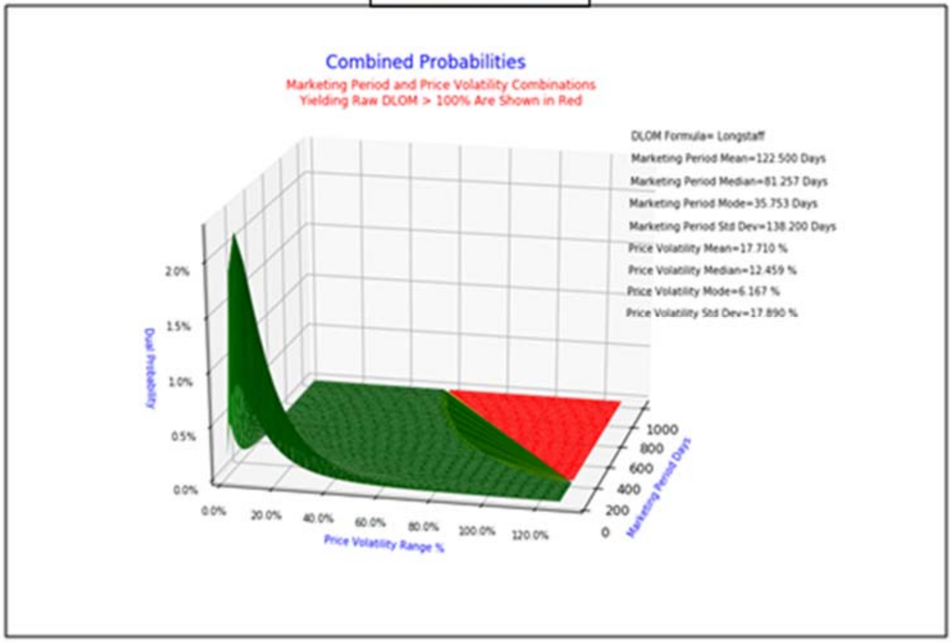
Figure 9.4 CVLY



Source: VFC DLOM Calculator, www.dlomcalculator.com

Why did FTGI have historical price volatility of 254.97% on September 25, 2007? We can surmise that it was because FTGI had negative preceding year EBITDA that was equal to its preceding year revenues, 150% of its book equity, and over 40% of its assets. Investors evidently agreed that FTGI was a very risky stock. That risk was appropriately reflected in very high stock price volatility. Figure 9.5 shows FTGI's marketing period and price volatility risk profiles based on the means and standard deviations in Table 9.3. Note the significant difference in the y axis of Figure scale and skewing of the probability lines compared to Figure 9.1—the Figure 9.5 y axis scale is an order of magnitude greater than the Figure 9.1 scale. This is caused by FTGI's much greater price volatility compared to CVLY's. But the marketing period x axis scale of Figure 9.5 is half of the Figure 9.1 marketing period scale. These comparisons help explain why the FTGI DLOM is 7.7 times the CVLY DLOM.

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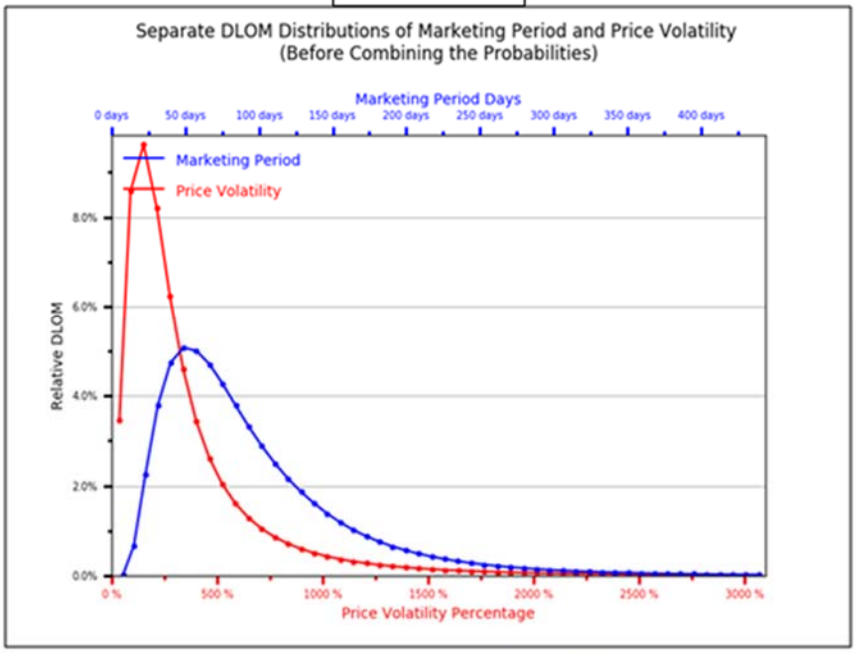
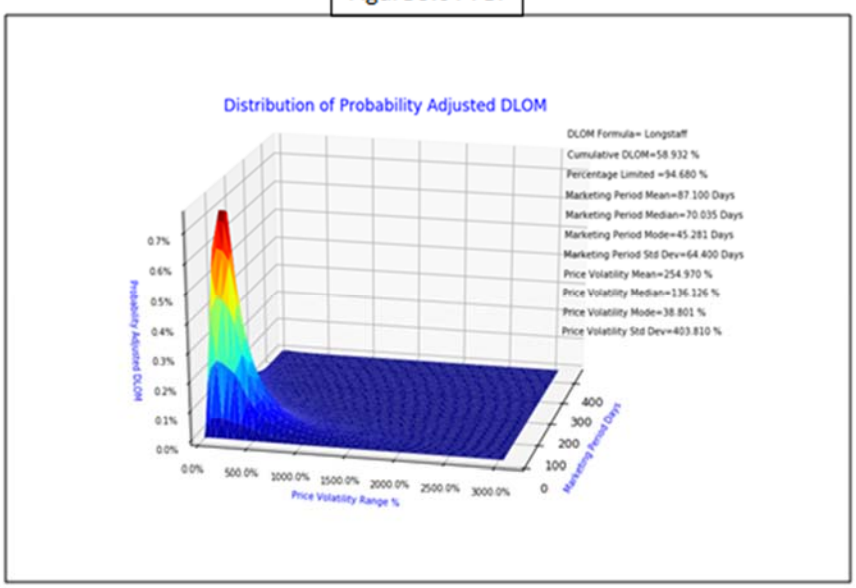
Figure 9.5 FTGI



Source: VFC DLOM Calculator, www.dlomcalculator.com

Figure 9.6 shows FTGI's double probability risk profile based on the combined risk profiles of Figure 9.5. The peaked area represents the most likely combined occurrences and accounts for the lion's share of FTGI's 58.93% double probability VFC Longstaff DLOM shown in Table 9.3.

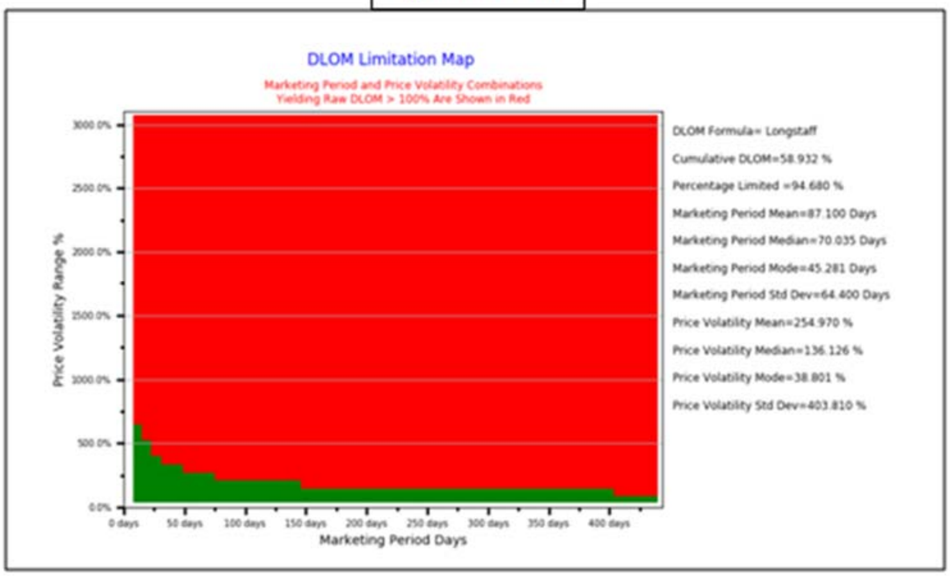
Figure 9.6 FTGI



Source: VFC DLOM Calculator, www.dlomcalculator.com

Figure 9.7 is a two-dimensional display that shows in red the proportion of FTGI's combined marketing period and price volatility probabilities that result in raw VFC Longstaff DLOMs of 100%. Note the significantly greater number of red combinations for FTGI compared those for CVLY shown in Figure 9.3.

Figure 9.7 FTGI



Source: VFC DLOM Calculator, www.dlomcalculator.com

Figure 9.8 is a three dimensional display that shows in red the low probabilities associated with FTGI's combinations of marketing period and price volatility probabilities that result in VFC Longstaff DLOMs of 100%. Note that the probability of red combinations remains very low. The high-probability area presented in green accounts for the lion's share of FTGI's 58.93% VFC Longstaff DLOM.

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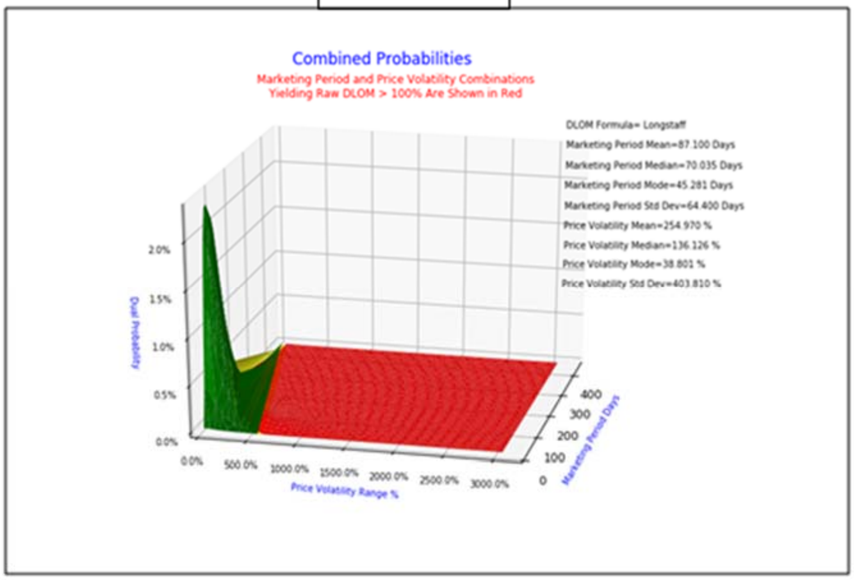
Figure 9.8 FTGI

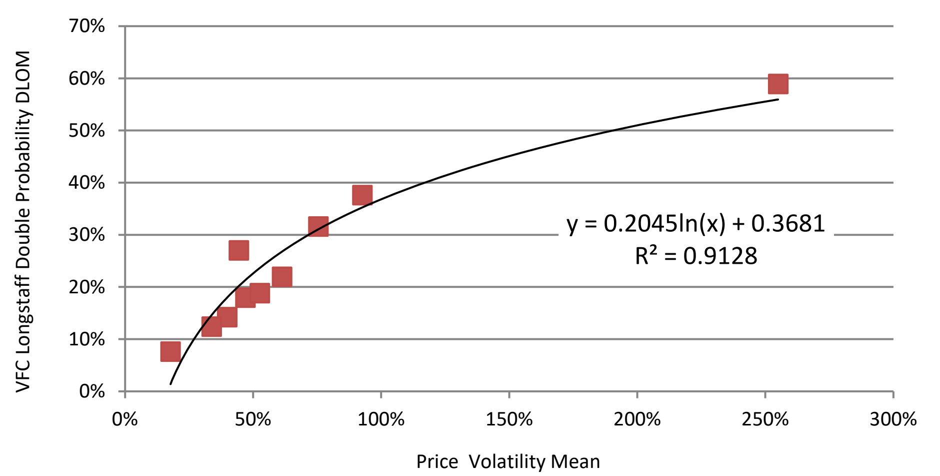


Source: VFC DLOM Calculator, www.dlomcalculator.com

Readers can consider whether the financial differences shown Tables 9.2 and 9.3 explain the relative price volatilities of the other stocks in the list. Figure 9.9 shows the correlation of the VFC double probability DLOMs and underlying average price volatilities per Table 9.3. The R-square of correlation of the average price volatilities with the VFC Longstaff DLOMs is 91.28%.

With the understanding that DLOMs should represent the investment risk, it can be seen that the VFC DLOM Calculator® yields DLOMs that are highly responsive to the risks of price volatility associated with the illiquid time periods required to sell investments.

Figure 9.9
Logarithmic Regression of Table 9.3 VFC Mean Price Volatilities and DLOMs



Chapter 10

REVISITING *"LITMAN AUDACITY"*

Pluris® president Espen Robak discussed the "wildly different discounts" presented to the Federal Circuit court in a well-written analysis of the tax case *Litman v. The United States*.[160] *Litman* in part dealt with the appropriate DLOM to apply to four tranches of Hotels.com restricted stock issued to David Litman on February 24, 2000, coincident with the Hotels.com initial public stock offering. Mr. Robak's article includes a table that summarized the conflicting DLOM opinions of the three valuation experts—one each for the IRS, Hotels.com, and Mr. Litman. Table 10.1, which restates the information summarized in the Robak article, shows each expert's DLOM opinion for each tranche of stock, and the court's corresponding conclusions:

Table 10.1
DLOM Estimates for HOTELS.COM Restricted Stock as of February 24, 2000

			As Presented to and Decided by the Court			
Tranche Restriction	# of Shares	Weight	The IRS	Hotels.com	Litman	The Court
One Year	1,959,960	20%	16.9%	20.0%	49.5%	22.0%
Two Years	489,990	5%	20.9%	20.0%	61.5%	36.0%
Three Years	489,990	5%	21.2%	20.0%	63.5%	38.0%
Four Years	7,059,960	70%	21.2%	20.0%	79.0%	50.0%
Total Shares	9,999,900					
Weighted DLOM		100%	20.3%	20.0%	71.3%	43.0%

Litman provides an opportunity to demonstrate how DLOMs calculated using the VFC DLOM Calculator® compare to those presented by the experts and decided by the Court. The following parameters were selected for this demonstration:

- Valuation date: February 24, 2000.
- Guideline stock: The Priceline Group, Inc. (PCLN).
- PCLN price volatility look-back period = We attempted to obtain price data for the 250 trading days prior to the valuation date, but PCLN's initial public offering occurred just 36 trading days before the valuation date.
- PCLN price volatility: Average = 58.1%. Standard deviation = 44.5%.
- Source of marketing period data:[161] VFC's *BIZCOMPS®* database for SIC code 7389. Transactions that closed during the 10 years prior to the valuation date.

[160] *Litman v. The United States*, Nos. 05-956T, 05-971T, 06-285T (August 22, 2007, modified March 20, 2008).

[161] This demonstration assumes that registration of Mr. Litman's shares was contractually prohibited, leaving only the possibility of a private sale. Therefore, the illiquidity time period is based on BIZCOMPS® data instead of the processing time to obtain SEC registration.

- Private sale marketing period probabilities: Average = 187.3 days. Standard deviation = 140.2 days. Number of transactions = 76. Confidence interval = 155.8 days to 218.9 days.
- Application of SEC Rule 144 subsequent to the lock-up periods is ignored.

Lognormal distribution of the above price volatility was used in combination with the time periods of the contractual restriction to calculate single probability DLOMs using the VFC Longstaff and VFC Black-Scholes formulas of the VFC DLOM Calculator®. The assumption is that each tranche would have been sold immediately at the end of its restriction period. Table 10.2 summarizes the calculated single-probability DLOMs.

Table 10.2

VFC DLOM Estimates for HOTELS.COM Restricted Stock as of February 24, 2000
Risk Free Rate = Zero; Dividend Rate = Zero

Single Probability 250-Day Pre-Closing Price Volatility
Marketing Period = Tranche Restriction

Tranche Restriction	VFC Longstaff	VFC Black-Scholes	The Court
One Year	50.7%	22.2%	22.0%
Two Years	65.4%	30.4%	36.0%
Three Years	73.6%	36.0%	38.0%
Four Years	79.0%	40.5%	50.0%
Weighted Average	72.3%	36.0%	43.0%

Table 10.2 shows that the single-probability VFC Black-Scholes DLOMs approximates the Court's tranche-based DLOM estimates, except that the Court's DLOM estimate for the four-year tranche is considerably higher than the single-probability VFC Black-Scholes estimate. The weighted average of the Court's DLOM estimates is 43% compared to the 36% weighted average using the single-probability VFC Black-Scholes method. The single-probability VFC Black-Scholes method appears to yield DLOM answers that the Court would have found to be reasonable and acceptable.

Comparing Table 10.2 to Table 10.1 reveals that the single-probability VFC Longstaff DLOMs very closely approximate the tranche-based opinions of Mr. Litman's expert, who used a CAPM method. The weighted average of Mr. Litman's expert's tranche-based DLOM estimates is 71.3% (see Table 10.1) while the single-probability VFC Longstaff approach results in a weighted average DLOM of 72.3% (see Table 10.2). Only the three-year tranche has substantially different DLOM estimates. The single-probability VFC Longstaff method appears to yield DLOM answers that Mr. Litman's expert would have found to be reasonable and acceptable.

The tranche conclusions of Mr. Litman's expert correlate almost perfectly with the Court's tranche conclusions. The R-square of that correlation is 98%. The individual tranche DLOMs of the single-probability VFC Longstaff formula also correlate highly with the court's tranche conclusions, with an R-square of 93%. Thus, the single-probability VFC Longstaff DLOMs are corroborated by the opinions of Mr. Litman's expert and the DLOM trends decided by the court.

Double-probability DLOMs are not tranche sensitive, because it is assumed that Mr. Litman could have sold all 9,999,900 shares of Hotels.com stock that he owned in a private transaction at any time. The period of time that it would take to find a private buyer and to consummate the sale represents the alternative period of illiquidity associated with Mr. Litman's stock—that is, the probability of outcomes defined by private sales of SIC code 7389 businesses. According to the VFC *BIZCOMPS®* database, this probability has a mean of about 194.6 days and a standard deviation of about 152.2 days.

The double-probability DLOM calculated using the VFC Longstaff formula is 34.5%, while the double-probability DLOM calculated using the VFC Black-Scholes formula is 15.1%. *See* Table 10.3. With a value approximately midway between the IRS's and Hotels.com's weighted average DLOMs on the one hand and the Court's weighted average DLOM on the other hand, it appears that the parties would have found the double probability VFC Longstaff DLOM result of 34.5% to be reasonable and acceptable.

Table 10.3

Double Probability 250-Day PCLN Price Volatility
Marketing Period = *BIZCOMPS®* SIC Code 7389

VFC Longstaff	VFC Black-Scholes
34.5%	15.1%

Chapter 11

Discounts for Lack of Marketability: What the Scientific Method Tells Us[162]

Observe. Hypothesize. Test. These are the principles of the scientific method. The scientific method seeks answers to questions raised from observation; formulates hypotheses based on the observations and questions; makes predictions regarding the hypotheses; and tests the hypotheses. A scientific hypothesis must be falsifiable to have a valid test, meaning that it must be possible to disprove the hypothesis. A hypothesis that is impossible to disprove is inherently unreliable, because it cannot be confirmed. Conversely, a hypothesis that is falsified is proved to be wrong.

Valuation practice regarding discounts for lack of marketability (DLOM) has historically relied on implied hypotheses that in and of themselves cannot be falsified. For example, practitioners may intuitively hypothesize that the following price difference observations represent DLOM because they may reflect a degree of lack of marketability:[163]

1. The differences between restricted stock prices and their contemporaneous unrestricted stock prices.
2. The differences between pre-IPO stock prices and their noncontemporaneous IPO stock prices.
3. The "cost of flotation," meaning the cost of achieving marketability by means of a public offering.
4. Long-term equity anticipation securities (LEAPS), which represent the percentage cost of acquiring protective puts for publicly traded stocks. The percentage cost is the put cost of a publicly traded stock divided by the stock price.

A hypothesis that any one of the above price observation sets does or does not predict DLOM is not falsifiable in isolation, because it cannot be shown that any of the observations

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[162] This chapter was first published in the September/October 2022 issue of *The Value Examiner, A Professional Development Journal for the Consulting Disciplines.* The table and graph numbers are in the context of the published article.

[163] The *International Glossary of Business Valuation Terms* defines "marketability" as "the ability to quickly convert property to cash at minimal cost," and "discount for lack of marketability" as "an amount or percentage deducted from the value of an ownership interest to reflect the relative absence of marketability." The *International Glossary of Business Valuation Terms* was adopted in 2001 by American Institute of Certified Public Accountants, American Society of Appraisers, Canadian Institute of Chartered Business Valuators, National Association of Certified Valuation Analysts, and The Institute of Business Appraisers.

Discount for Lack of Marketability—Job Aid for IRS Valuation Professionals, September 25, 2009, page 5, explains the difference between "liquidity" and "marketability": "If it's liquid, it's marketable; If it's non-marketable, it's illiquid; Being illiquid does not [necessarily] mean non-marketable—it may still be sellable but not quickly or without loss of value."

represent DLOM and only DLOM.[164] We simply do not know what other ingredients influenced the price differentials, and we do not know, in isolation, what DLOM should be.

If the above-listed observations represent DLOM, they should yield statistically correlated results among themselves via linear regression, assuming that the transactions can be matched. Anecdotally they seem not to—practitioners usually produce very different results from the different transactional sources. If the linear regressions of any two of the different discount observations sufficiently correlate, then one could perhaps conclude that they are somehow related and predict DLOM. But the distinct types of transactions—with different measurement of the underlying transactions coupled with known and unknown ingredients, such as premiums, compensation, relationship biases, changes in holding periods, the size of the stock block, and changes in economic conditions, among others—suggest that the observed results will not correlate to a reasonable degree of statistical reliability.

It would be pointless to compare the discounts of the rare transactions that overlap the Pluris and FMV/Stout databases; any discount difference would be due to discount measurement and would tell us nothing about DLOM measurement. Meanwhile, regression analysis cannot be done for restricted stock transactions versus pre-IPO transactions versus flotation costs versus LEAPS transactions unless the opposing transactions can be matched—a seemingly impossible task. The different price observations of restricted stock discounts, pre-IPO transactions, cost of flotation, and LEAPS therefore contradict their general use for benchmarking a reliable DLOM for a specific valuation subject.[165]

The Benchmarking Methodologies

It is problematic for the valuation community that a positive hypothesis favoring DLOM benchmarking based on the above price observations can easily be falsified. The price differences of the benchmarked transactions can usually be shown to include known ingredients that inherently are not DLOM. Any such showing falsifies the hypothesis. Meanwhile, the null hypothesis (i.e., that the benchmarking does *not* represent DLOM) cannot be falsified without knowing theoretically correct DLOM numbers. The result is that benchmarking based directly on

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[164] Other price difference observations such as those by Bajaj and Abbott have been suggested to represent a lack of liquidity, but not necessarily to represent DLOM.

[165] An interesting study would be to attempt comparison of contemporaneous LEAPS percentage costs to restricted stock percentage discounts of the same stock. The result might falsify hypotheses that one does or does not predict the other. I have not made this study. I invite other researchers to undertake the task and to publish their results.

restricted stocks, pre-IPOs, flotation costs, or LEAPS should be considered unreliable conjecture.[166]

Additionally confounding the use of the available transactional databases for DLOM benchmarking is the limited number of transactions that might closely approximate the valuation subject. You can read about these limitations in my book, *Empirical Research Regarding Discounts for Lack of Marketability*, available free at https://dlomcalculator.com. Chapters 4 and 5 pertain to the restricted stock transactions reported by Pluris and FMV/Stout, respectively, and show the general lack of statistical significance of the relationship of the database companies' metrics and the observed transactional discounts.[167]

The Mandelbaum Criteria

The U.S. Tax Court uses the *Mandelbaum*[168] criteria to assess the reasonableness of DLOM estimates. The affirmation or rejection of a separately developed DLOM conclusion is an entirely acceptable use of the criteria. But the criteria do not directly yield a DLOM percentage. Using the criteria to directly estimate a DLOM is guessing. For that reason, a hypothesis that the *Mandelbaum* criteria directly yield a reliable DLOM is not falsifiable, and is an inherently unreliable use.

The Calculator Methodologies

Many practitioners intuitively hypothesize that other methodologies result in reasonable DLOM estimates, including:

1. The Quantitative Marketability Discount Model (QMDM), which is a spreadsheet methodology.
2. Calculators based on pre-IPO transactions.
3. The Pluris and FMV/Stout calculators, which are based on their databases of restricted stock transactions.

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[166] This deficiency is not ameliorated by using an "all of the above," "salad bowl" approach to DLOM estimation: garbage in, garbage out.

[167] *Empirical Research* r
eports that the reported restricted stock discounts most closely correlate with the price volatility of the issuers' publicly traded stocks. *See* pages 43, 46, 49, and 50 re Pluris; pages 88–92 re FMV/Stout.

[168] Mandelbaum v. Commissioner, T.C. Memo 1995-255 (U.S. Tax Ct. 1995).

4. Options-based formulas, such as Finnerty, Black-Scholes, and Longstaff, among others.
5. Probability-based DLOM, using the Finnerty, Black-Scholes, or Longstaff formula in conjunction with the VFC DLOM Calculator.[169]

Let us apply the discussion of the preceding paragraphs to these methodologies:

- The hypothesis to scientifically test QMDM would be, "The QMDM methodology predicts (or does not predict) the [what?] underlying discounts." Neither hypothesis can be scientifically tested because QMDM is not based on a set of transactions or data from which independent DLOM conclusions are derived. DLOM calculations are the "educated" guess of a single person deemed to have better judgment than all others. This makes QMDM an unreliable means of calculating DLOM.
- The pre-IPO, Pluris, and FMV/Stout calculators have underlying transaction databases against which their calculators can be statistically tested. Ignoring that the price differences may be (are?) due to more than DLOM, these calculators should predict the underlying observations. This would be the start of a scientific test.
- Proving that the Finnerty, Black-Scholes, and Longstaff formulas yield reliable DLOMs is not possible in isolation. The formulas are theoretical models that do not draw from actual transactions. More is needed to test them scientifically. It should be understood, however, that only the Longstaff formula was created to predict DLOM. The others were created to price options for investment purposes.

A hypothesis to test the database and formula DLOM calculation methodologies (excluding QMDM) is, "The [specified] DLOM methodology predicts the corresponding [specified] observed discounts." This hypothesis is conceptually falsifiable, but defective. First, the hypothesis impliedly assumes that the observed discounts represent DLOM. As discussed, that assumption is easily falsified. Second, a linear regression with an R-square of 1 percent offers a prediction that may be statistically measurable but leaves 99 percent of the correlation unexplained. It therefore makes sense to specify a minimum regression standard for falsifying the hypothesis.

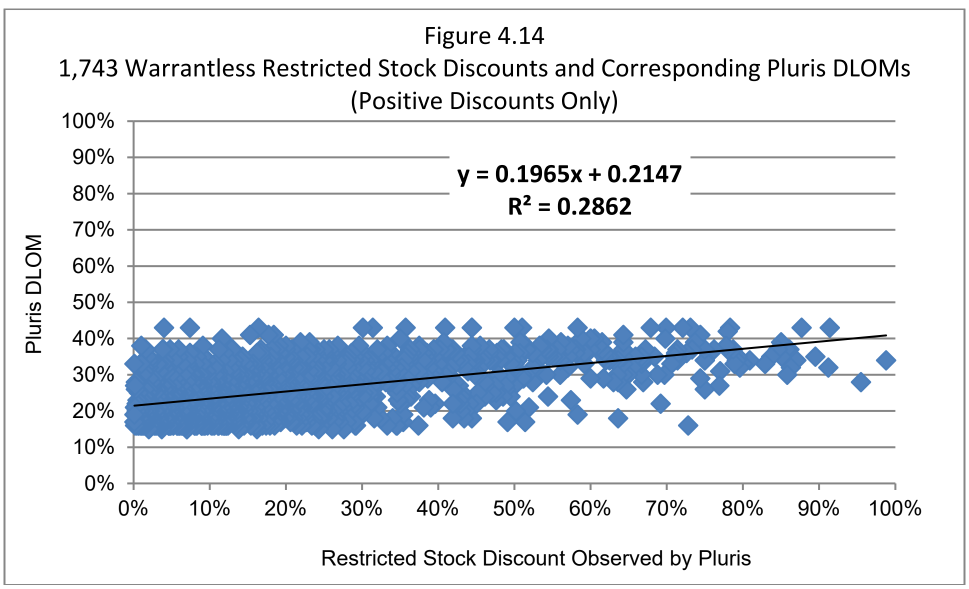
Setting aside the implication that the observed discounts necessarily represent DLOM, a more tightly crafted hypothesis is, "The [specified] methodology predicts the corresponding [specified] observed discounts with a linear regression relationship exhibiting these

[169] The VFC DLOM Calculator is located at https://dlomcalculator.com.

characteristics: (1) X-Y intercept ~ zero; (2) coefficient of x ~ y; and (3) R-square > 50 percent." This parameter-based hypothesis is clearly falsifiable, but is still not sufficient because a single negative result does not preclude the possibility of false positives that satisfy the parameters. The better hypothesis is stated in the negative (the "null hypothesis"): "The [specified] methodology *does not* predict the corresponding [specified] observed discounts with a linear regression relationship exhibiting these characteristics: (1) X-Y intercept ~ zero; (2) coefficient of x ~ y; and (3) R-square > 50 percent." A single result satisfying the parameters conclusively falsifies the null hypothesis even if contradictory results are found. Multiple tests falsifying the null hypothesis add substantiation to the conclusion that the method predicts the result.

A. The Database Calculators

Chapters 4 and 5 of *Empirical Research* discuss correlation tests performed on the Pluris and FMV/Stout calculator results with their respective databases. Excluding the discount-distorting restricted stock transactions with price premiums or warrants (premiums clearly are not DLOM; the warrants distort discount measurement), *Empirical Research* Figure 4.14 shows the weak regression results of the Pluris DLOM calculator versus the corresponding observed discounts of its database. I could not find a legitimate set of Pluris calculator DLOMs and corresponding restricted stock discounts that falsified the null hypothesis.[170]



[170] It would be possible to select a bottom-up set of transactions with discounts within the range of the Pluris (or FMV/Stout) calculator limitations to achieve satisfactory correlation results. This would be illegitimate, however, because the results would have been manufactured. A legitimate test requires top-down selection.

Table 1 (see below) reports the weak correlation of the FMV/Stout calculator versus the observed discounts of the corresponding 638 transactions. Again, I was unable to negate the null hypothesis. Only a group of eighteen and a group of five transactions were found to have statistical correlation greater than 50%, and only the group of five had desirable formula attributes with a line slope approximating 1.0 and the X-Y intercept approximating 0%. But the perfect correlation of the five-transaction group is excludable because it merely proves the regression methodology—any group of five divided into quintiles will result in a perfect correlation.

Table 1
Correlation of Median Quintile DLOMs with Positive Discount Transactions

Rule 144 Time Period	Registration Rights	Transaction Count	Average Transaction Discount	Average Stout DLOM	Regression Line Slope	Y-Axis Intercept	R-Square
All	All	638	21%	18%	1.926	-13.5%	26%
2 Years	No	178	24%	22%	1.714	-14.1%	29%
1 Year	Yes	159	17%	14%	2.420	-16.5%	30%
1 Year	Blank	121	27%	22%	2.067	-18.3%	25%
6 Months	Yes	94	15%	12%	4.230	-34.3%	15%
2 Years	Yes	40	24%	24%	3.391	-56.0%	45%
1 Year	No	23	25%	23%	1.436	-7.9%	22%
6 Months	No	18	17%	17%	2.155	-19.5%	60%
2 Years	Blank	5	23%	23%	1.000	0.0%	100%

These test results undermine using the Pluris and FMV/Stout calculators to reach reliable DLOM conclusions. The tests show that the calculators do not falsify the null hypothesis, and do not replicate the underlying observations. It may be that tests performed by other persons will find different results, but until then one must conclude that there is no scientific basis for stating that the calculators reliably predict the underlying transactions even if one were to accept, erroneously, that the populations of observed discounts represents DLOM.

I did not perform similar analyses of pre-IPO calculators and their underlying transactions using the null hypothesis. Others are invited to do so and to publish the results.

B. The Formulas and Formula Calculators

I used three forms of simulation-derived data to test the Finnerty, Black-Scholes, and Longstaff formulas. First, we used a cleansed population of restricted stock discounts derived from the Pluris and FMV/Stout restricted stock databases. The cleansing process is discussed below. Second, we obtained the daily stock closing prices for the restricted stock issuers for time periods ending with the date the restricted stocks were issued. I calculated and considered price volatility means and standard deviations for a variety of pre-issue time periods as discussed below. Third, I measured the time it took the SEC to approve S-1 filings on a standard industrial classification basis. The derived information allowed us to scientifically address the Finnerty, Black-Scholes, and Longstaff formulas using the null hypothesis, "The [specified] formula *does not predict* the corresponding observed restricted stock discounts with a linear regression relationship exhibiting these characteristics: (1) X-Y intercept ~ zero; (2) coefficient of X ~ Y; and (3) R-square > 50%." I refer to this as the "formulas null hypothesis." The Finnerty and Black-Sholes formulas include a risk-free rate parameter, and the Black-Scholes formula includes a dividend yield parameter. The tests discussed in this article assume zero for these parameters. See *Empirical Research*, Chapter 7, for more discussion of the zero assumptions.

- I tested the formulas with the static price volatilities reported by Pluris and FMV/Stout and 90, 180, and 360 days as static time variables. None of the R-squares of correlation exceeded 6.5% using the Pluris and FMV/Stout population of 4,372 transactions for which the databases reported price volatility. The formulas null hypothesis was therefore not falsified.
- Zero or negative discounts are reported for many of the transactions in the Pluris and FMV/Stout databases. Removing those reduced the population to 3,869 transactions. I performed the same 90, 180, and 360-day marketing period tests of this populations. The range of R-squares of correlation for these tests was from 18.32% to 25.38%. The formulas null hypothesis was again not falsified.

The preceding paragraph shows that removing statistical noise has a positive effect on the regression results. Further, the other non-DLOM characteristics affect the restricted stock discounts reported by Pluris and FMV/Stout. For example, the warrant-tainted stocks in the Pluris database distort the reported transaction discounts. These and other transaction characteristics can cause the X-Y intercept to deviate from zero—a widespread problem in empirical research. Culling the population of restricted stock transactions in the databases to remove such statistical noise is necessary. Table 2 shows the initial culling process based on (a) the Pluris and FMV/Stout reported price volatilities, and (b) the recalculated mean price volatilities based on available reported daily closing prices.

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Table 2

Population Refinement Improves the Relationship of Restricted Stock Discounts to Price Volatilities [171]

	FMV/Stout and Pluris® Reported Volatilities		VFC DLOM Calculator® Average Price Volatilities	
	Number of Transactions	Logarithmic Regression	Number of Transactions	Logarithmic Regression
All Stout Study (769) and Pluris® (3,632) restricted stock transactions	4,401		4,401	
Transactions with no price volatility reported by Pluris® and Stout	(29)		n/a	
	4,372	R² = 0.0622	4,401	
Pluris® transactions with warrants reported	(1,867)		(1,867)	
	2,505	R² = 0.0384	2,534	
Transactions closing dates prior to September 15, 2007 (price history not available)	n/a		(1,687)	
	2,505	R² = 0.0384	847	
Issuers apparently no longer publicly traded	n/a		(427)	
	2,505 [172]	R² = 0.0384	420	
Issuers with zero percent price volatility	n/a		(13)	
	2,505	R² = 0.0384	407 [173]	R² = 0.0182
FMV/Stout duplicates for which Pluris® has warrants	(2)		(1)	
	2,503	R² = 0.0384	406	R² = 0.018
Pluris® transactions with FMV/Stout duplicate (priority was given to FMV/Stout transactions)	(196)		(48)	
	2,307 [174]	R² = 0.0359	358 [175]	R² = 0.0144
Transactions with zero or negative discounts	(382)		(91)	
Positive discount transactions with price volatilities (excludes duplicates)	1,925	R² = 0.2348	267	R² = 0.0898
Issuer stock prices that failed VFC's price verification test	n/a		(67)	
Refined restricted stock issuer dataset	1,925 [176]	R² = 0.2348	200 [177]	R² = 0.179

[171] The "FMV/Stout and Pluris" columns use the one-year price volatilities for the transactions as reported by those databases. The "VFC DLOM Calculator" columns use the restricted stock issuer's average price volatilities calculated by the VFC DLOM Calculator for the 250 days preceding the applicable transaction closing date.

[172] Using linear regression, this group of transactions has a t Stat of 8.8 and a P-value of 2.8E-18. The relationship is statistically significant.

[173] Using linear regression, this group of transactions has a t Stat of 2.5 and a P-value of 0.0127. The relationship is statistically significant.

[174] Using linear regression, this group of transactions has a t Stat of 8.2 and a P-value of 4.0E-16. The relationship is statistically significant.

[175] Using linear regression, this group of transactions has a t Stat of 2.1 and a P-value of 0.0345. The relationship is statistically significant.

[176] Using linear regression, this group of transactions has a t Stat of 17.9 and a P-value of 3.2E-66. The relationship is statistically significant.

[177] Using linear regression, this group of transactions has a t Stat of 7.9 and a P-value of 1.7E-13. The relationship is statistically significant.

Table 3 shows the second round of culling based on (1) the registration rights reported by Pluris and FMV/Stout; (2) the stock issuers' SIC codes; and (3) excluding the years of and before the Great Recession. The combined culling resulted in a population of 59 "clean" restricted stock transactions. Table 3 shows the improvement in DLOM regression results as the dataset was refined.

Table 3
Secondary Refinement of the Test Dataset

					Linear Regressions v Transaction Discounts					
					VFC Longstaff DDLOM			VFC Black-Scholes DDLOM		
Number of Restricted Stock Transactions	Closing Date Range	Number of SEC Approvals in the Issuers' 4-Digit SIC Codes	Transaction Discount	Registration Rights	Slope	Intercept	R-Square	Slope	Intercept	R-Square
Refined Restricted Stock Issuer Dataset with VFC Calculated Price Volatility Probabilities										
200 per Table 2	2007-2014	n/a	> 0%	DR, MR, NR, PB, No, Yes, and Unknown	DLOMs could not be calculated for 55 transactions because the issuers' reported 4-digit SIC code could not be found in the VFC database of SEC filings.					
R-Squares of Correlation and Regression Formulas Improve with More Specific SIC Codes; When Transactions with Unknown Registration Rights Are Removed; and When the Great Recession Years Are Removed										
145	2007-2014	1 or more	> 0%	DR, MR, NR, PB, No, Yes, and Unknown	0.7520	5.20%	19.93%	1.6415	4.86%	23.79%
140	2007-2014	2 or more	> 0%	DR, MR, NR, PB, No, Yes, and Unknown	0.7738	4.29%	21.23%	1.6872	3.95%	25.41%
130	2007-2014	3 or more	> 0%	DR, MR, NR, PB, No, Yes, and Unknown	0.8334	3.35%	24.28%	1.8037	3.10%	28.77%
118	2007-2014	4 or more	> 0%	DR, MR, NR, PB, No, Yes, and Unknown	0.8984	2.15%	26.94%	1.9796	1.61%	31.98%
75	2010-2014	4 or more	> 0%	DR, MR, NR, PB, No, Yes, and Unknown	1.0612	-0.19%	35.86%	2.2480	-0.29%	41.49%
59	2010-2014	4 or more	> 0%	DR, MR, NR, PB, No, Yes	1.0109	-2.84%	54.19%	2.0769	-2.08%	57.45%

Returning briefly to subsection A, above, the FMV/Stout and Pluris calculator DLOMs were retested using this 59-transaction population. The calculator results had only a 16.1 percent R-square of correlation with the observed discounts of the transactions, again failing to falsify their null hypotheses.

Continuing with the Finnerty, Black-Sholes, and Longstaff formulas, these were tested using the 59-transaction population; the price volatilities reported by Pluris and FMV/Stout; and 90, 180, and 360-day static marketing periods (see Table 4).[178] These tests were statistically significant. However, the X-coefficients of the regression lines failed to approximate 1.0, and the highest R-square of correlation was 42.88 percent. The tests failed to falsify the formulas null hypothesis.

Table 4
Regression Results for 59 Pluris and FMV/Stout Restricted Stock Transactions
Finnerty, Black-Scholes, and Longstaff Formula DLOMs[179]
Using the Database Price Volatilities[180] and 90, 180 and 360 Days to Sale Date

	Formula	R-Square
Finnerty: Database volatility, StdDev = 0; Marketing mean 90 days, StdDev = 0	y = 3.2548x + 0.0018	R^2 = 0.4193
Black-Scholes: Database volatility, StdDev = 0; Marketing mean 90 days, StdDev = 0	y = 0.8346x + 0.0134	R^2 = 0.4288
Longstaff: Database volatility, StdDev = 0; Marketing mean 90 days, StdDev = 0	y = 0.3797x + 0.0092	R^2 = 0.3954
Finnerty: Database volatility, StdDev = 0; Marketing mean 180 days, StdDev = 0	y = 1.9218x - 0.0158	R^2 = 0.3920
Black-Scholes: Database volatility, StdDev = 0; Marketing mean 180 days, StdDev = 0	y = 0.6389x + 0.0054	R^2 = 0.4235
Longstaff: Database volatility, StdDev = 0; Marketing mean 180 days, StdDev = 0	y = 0.3019x + 0.0010	R^2 = 0.2871
Finnerty: Database volatility, StdDev = 0; Marketing mean 360 days, StdDev = 0	y = 1.1856x - 0.0362	R^2 = 0.3314
Black-Scholes: Database volatility, StdDev = 0; Marketing mean 360 days, StdDev = 0	y = 0.5140x - 0.0080	R^2 = 0.4092
	y = 0.2362x + 0.0023	R^2 = 0.1691

[178] See Exhibit 1 for the transaction details and regression statistics. Exhibit 1 and the other exhibits referred to below can be found at **[LINK TO COME].**

[179] As applicable, the risk-free rate and dividend yield variables in the Black-Scholes and Finnerty formulas are assumed to be zero. A criticism of the Longstaff formula as conventionally applied is that it can yield values greater than 100 percent, with high volatility and long time-period estimates. People have characterized the formula as "breaking down." The criticism is invalid because no one has shown that the formula actually "breaks." But since DLOM cannot be greater than 100 percent, the logical solution is to limit Longstaff DLOM results to 100 percent. That approach is taken in this article when applying the described probability-based analysis. This topic is discussed fully in *Empirical Research,* chapter 7.

[180] The Pluris and FMV/Stout databases do not provide the standard deviations of the issuers' stock price volatilities.

Longstaff: Database volatility, StdDev = 0; Marketing mean 360 days, StdDev = 0

I then reconsidered the 59-transaction population using 250 (or the maximum available) trading days of average price volatility preceding each transaction closing date, and the assumed 90, 180, and 360-day marketing periods. Average price volatility was calculated using the VFC DLOM Calculator. The standard deviation of price volatility was ignored (see Table 5).[181]

Table 5
Regression Results for 59 Pluris and FMV/Stout Restricted Stock Transactions
Finnerty, Black-Scholes, and Longstaff Formula DLOMs
Using VFC-Calculated 250-Day Average Price Volatilities and 90, 180 and 360 Days to Sale Date

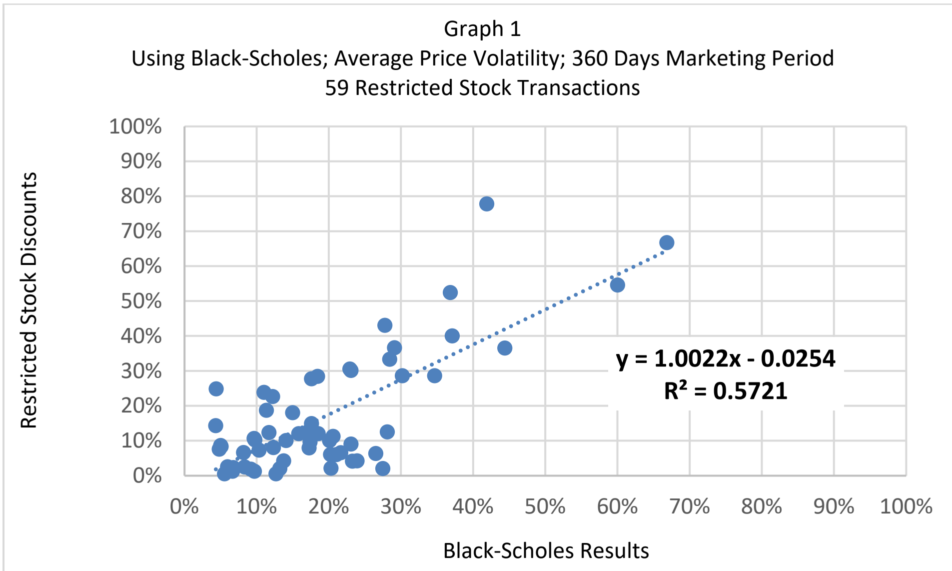
	Formula	R-Square
Finnerty: PV 250 days NASDAQ mean, StdDev = 0; Marketing mean 90 days, Std Dev = 0	y = 6.7043x - 0.0200	R^2 = 0.5751
Black-Scholes: PV 250 days NASDAQ mean, StdDev = 0; Marketing mean 90 days, Std Dev = 0	y = 1.8632x - 0.0159	R^2 = 0.5762
Longstaff: PV 250 days NASDAQ mean, StdDev = 0; Marketing mean 90 days, Std Dev = 0	y = 0.7149x + 0.0084	R^2 = 0.5804
Finnerty: PV 250 days NASDAQ mean, StdDev = 0; Marketing mean 180 days, Std Dev = 0	y = 3.5591x - 0.0280	R^2 = 0.5707
Black-Scholes: PV 250 days NASDAQ mean, StdDev = 0; Marketing mean 180 days, Std Dev = 0	y = 1.3517x - 0.0193	R^2 = 0.5752
Longstaff: PV 250 days NASDAQ mean, StdDev = 0; Marketing mean 180 days, Std Dev = 0	y = 0.5836x - 0.0162	R^2 = 0.5917
Finnerty: PV 250 days NASDAQ mean, StdDev = 0; Marketing mean 360 days, Std Dev = 0	y = 1.9865x - 0.0427	R^2 = 0.5526
Black-Scholes: PV 250 days NASDAQ mean, StdDev = 0; Marketing mean 360 days, Std Dev = 0	y = 1.0022x - 0.0254	R^2 = 0.5721
Longstaff: PV 250 days NASDAQ mean, StdDev = 0; Marketing mean 360 days, Std Dev = 0	y = 0.4409x - 0.0278	R^2 = 0.5157

Table 5 shows that these tests resulted in R-squares of correlation ranging from of 51.57 percent to 59.17 percent. Eight of the regression results had X-Y intercepts ranging from positive 0.84 percent to minus 2.8 percent.[182] These results meet two of the parameters for potentially falsifying the formulas null hypothesis. However, only one test successfully falsified the hypothesis with all three parameters: Black-Scholes using 360 days as the marketing period

[181] *See* Exhibit 2 **(LINK TO COME)** for the VFC-calculated price volatility means and standard deviations; *see* Exhibit 3 **(LINK TO COME)** for the calculated DLOMs and regression statistics of these tests. All were statistically significant.

[182] The ninth test had an X-Y intercept of minus 4.27 percent. I find this too distant from "X-Y intercept ~ zero."

assumption, shown in bold type in Table 5. This test resulted in a coefficient of X of 1.0022, an X-Y intercept of -2.54 percent, and an R-square of correlation of 57.21 percent[183] (see Graph 1). It is error to say that the Black-Scholes formula with 0 percent assumed for the risk-free rate and the dividend yield variables, 250 preceding trading days of price volatility, and an assumed 360-day time period, did not reliably predict the set of corresponding restricted stock discounts.



Adding probability to the time and price volatility variables of the Finnerty, Black-Scholes, and Longstaff formulas makes a material difference in their results.[184] The crux of this concept is that we do not live in a static world. Instead, everything is constantly changing, which applies to the price volatility and marketing period variables of the three formulas. It is therefore illogical to use constant values for those variables when trying to calculate the DLOM applicable to an investment that does not have a fixed price or a fixed time period to the liquidity event. As discussed in *Empirical Research*, I surmised that probability could be built into the formulas' DLOM results by determining both the mean and standard deviation of the price volatility and time period variables; spreading the means over the range of outcomes (the VFC DLOM Calculator assumes 50 statistical buckets determined by the standard deviations); calculating a separate DLOM for each combination of price volatility and time (marketing period); applying the probability of occurrence to each datapoint (totaling 100 percent); and summing the resulting DLOMs. Fifty price volatility points times fifty marketing period points is 2,500 datapoints. The distributions of

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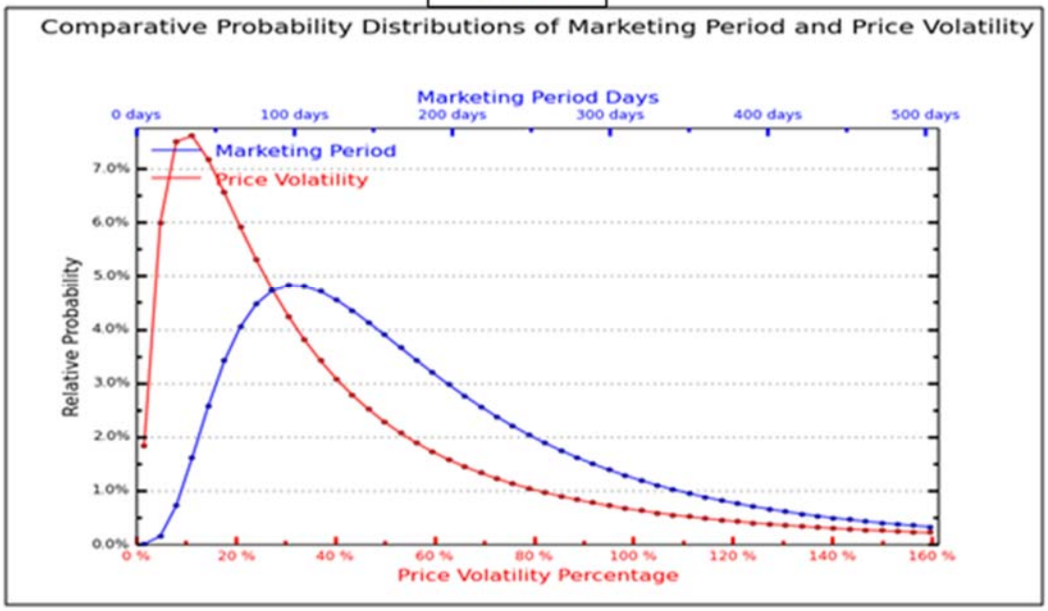
[183] The eight other tests had coefficients of X at or below 0.7149 or at or above 1.3517. I find these too distant from "coefficient of X ~ Y."

[184] See *Empirical Research,* chapters 6, 7, and 8.

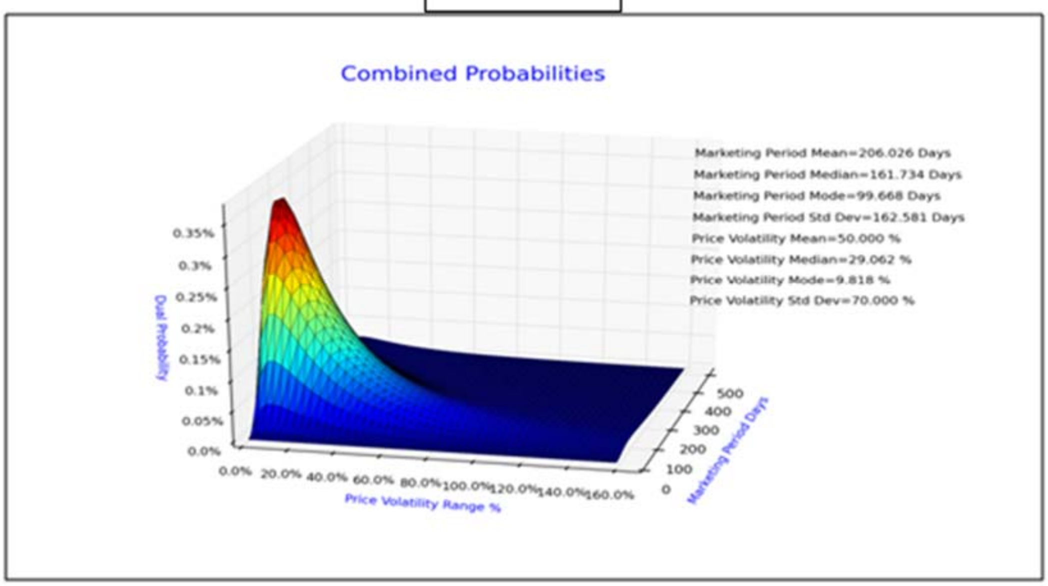
price volatility and time might separately look like *Empirical Research* Figure 6.22. Combined, those distributions look like *Empirical Research* Figure 6.23.

Figure 6.22



Source: VFC DLOM Calculator, www.dlomcalculator.com

Figure 6.23



Source: VFC DLOM Calculator, www.dlomcalculator.com

The cone in Figure 6.23 represents the combinations of price volatility and time that are most likely to occur, but any combination shown in the graph has a chance of occurrence. The entire area of the graph represents 100 percent of the potential outcomes. Calculating DLOM using the Figure 6.23 concept results in probability-based calculations using the Finnerty, Black-

Scholes, and Longstaff formulas. As the means and standard deviations of the price volatility and time source data change, so too does the distribution of the combined probability.

I also surmised that the expected time to obtain SEC approval for the restricted stock transaction might have affected the observed discounts. Accordingly, in addition to the mean and standard deviation of the issuers' closing stock prices for the 250 trading days preceding the closing date of each stock transaction, the means and standard deviations of the SEC approval periods for the issuers' 1- to 4-digit SIC codes for up to 10 years preceding the closing dates were calculated using the VFC DLOM Calculator. DLOMs were then calculated for each formula, using (a) only average price volatility and only average SEC processing time; (b) only the average price volatility, but probability-based SEC processing times; and (c) probability-based price volatility, but only average SEC processing time. The tests were run using the 4-digit SIC code means and standard deviations as described in Table 6.[185] These tests resulted in R-squares of correlation ranging from 54.66 percent to 62.81 percent. The X-Y intercepts of these tests ranged from positive 0.58 percent to negative 3.02 percent.[186]

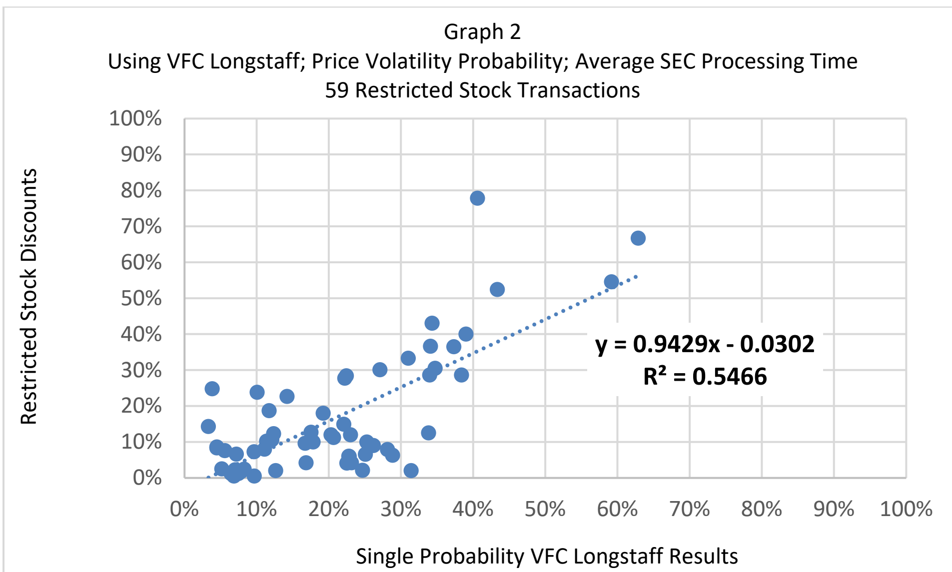
Table 6
Regression Results for 59 Pluris and FMV/Stout Restricted Stock Transactions
Using Combinations of VFC-Calculated Price Volatilities and SEC Processing Times with/without Probabilities

	Formula	R-Square
Finnerty: PV 250 NASDAQ mean STDDEV = 0; SEC processing 4-digit SIC code mean, StdDev = 0	y = 5.1829x - 0.0047	R^2 = 0.608
Black-Scholes: PV 250 NASDAQ mean STDDEV 0; SEC processing 4-digit SIC code mean, StdDev = 0	y = 1.6943x - 0.0131	R^2 = 0.6133
Longstaff: PV 250 NASDAQ mean STDDEV 0; SEC processing 4-digit SIC code mean, StdDev = 0	y = 0.6658x + 0.0058	R^2 = 0.6281
Finnerty: PV 250 NASDAQ mean STDDEV 0; Marketing Period Probability 4-digit SIC code	y = 5.4097x - 0.0068	R^2 = 0.5997
Black-Scholes: PV 250 NASDAQ mean STDDEV 0; Marketing Period Probability 4-digit SIC code	y = 1.7994x - 0.0111	R^2 = 0.6036
Longstaff: PV 250 NASDAQ mean STDDEV 0; Marketing Period Probability 4-digit SIC code	y = 0.7467x + 7E-05	R^2 = 0.6185
Finnerty: Price Volatility Probability; SEC processing 4-digit SIC code mean, StdDev = 0	y = 6.3251x - 0.0168	R^2 = 0.5501
Black-Scholes: Price Volatility Probability; SEC processing 4-digit SIC code mean, StdDev = 0	y = 1.9526x - 0.0231	R^2 = 0.5835
Longstaff: Price Volatility Probability; SEC processing 4-digit SIC code mean, StdDev = 0	y = 0.9429x - 0.0302	R^2 = 0.5466

185 *See* Exhibit 2 **(LINK TO COME)** for the details.

186 *See* Exhibit 4 **(LINK TO COME)** for the calculated DLOMs and regression statistics of these tests. All were statistically significant.

The Table 6 R-square and intercept ranges reasonably meet two of the parameters of the formulas null hypothesis. However, only one test reasonably satisfied the coefficient of X parameter: VFC Longstaff using probability-based price volatility and average (i.e., static) SEC processing times for the applicable SIC codes (shown in bold type in Table 6). This test resulted in a coefficient of X of 0.9429, an X-Y intercept of 3.02 percent, and an R-square of correlation of 54.66 percent (see Graph 2). This test reasonably falsified the formulas null hypothesis, although I prefer that the X coefficient be within +/– 5 percent of 1.0. With that caveat, it is error to say that the VFC Longstaff method using company-specific price volatility probabilities and industry-specific SEC average processing times did not predict the set of corresponding restricted stock discounts.[187] Because the Longstaff formula was crafted specifically to estimate DLOM, it would also be error to say that this method did not reliably predict DLOM.



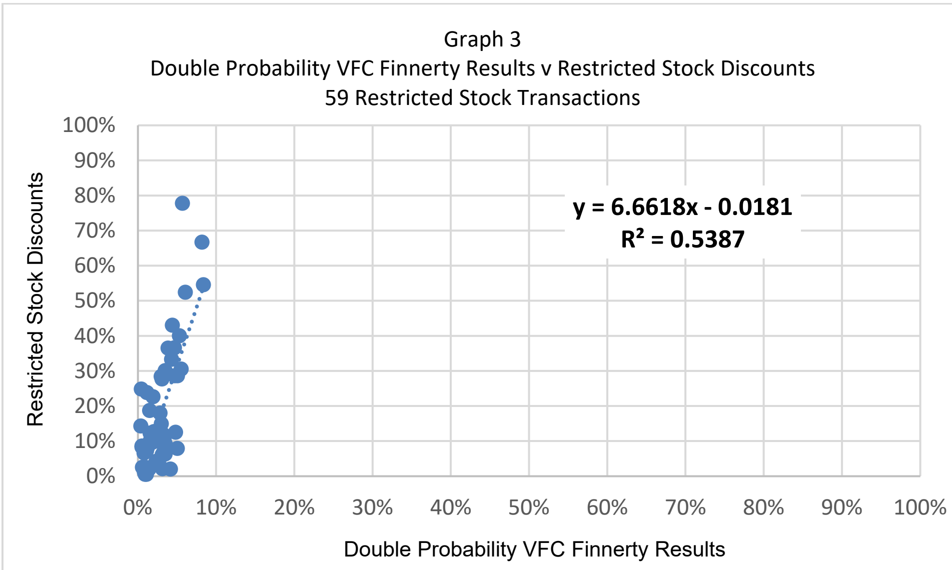
Finally, I considered double-probability calculations of the Finnerty, Black-Scholes, and Longstaff formulas, meaning that the tests were performed using the means and standard deviations of both price volatility and SEC processing time pursuant to the Figure 6.23 concept.[188] Although their R-squares of correlation and X-Y intercepts were acceptable, the Finnerty and Black-Scholes formulas failed their tests of the formulas null hypotheses with coefficients of X equal to 6.6618 (Graph 3) and 2.0769 (Graph 4), respectively. These coefficients mean that the

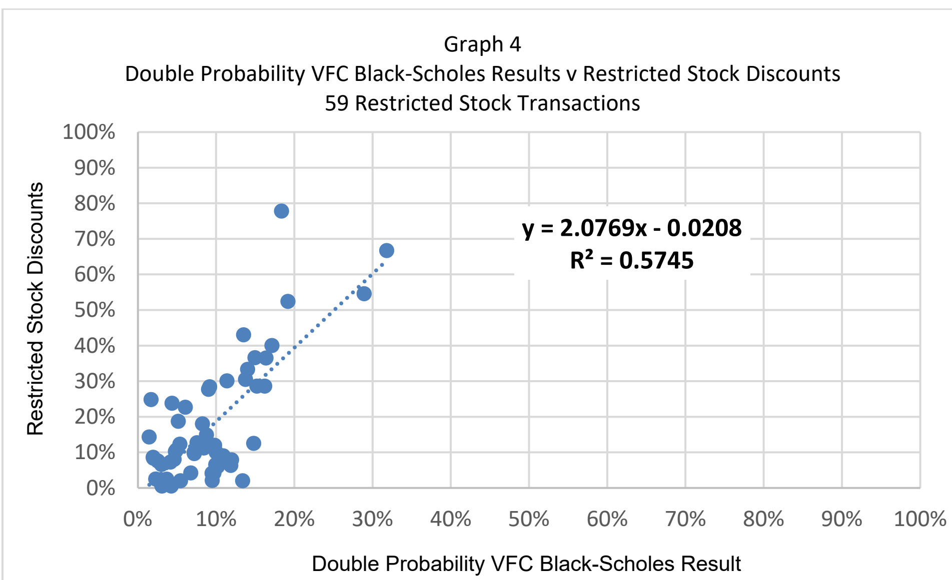
[187] The VFC Longstaff method limits Longstaff formula DLOMs to 100 percent. See *Empirical Research,* Chapter 7 for more information.

[188] *See* Exhibit 5 **(LINK TO COME)** for the calculated DLOMs and regression statistics of these tests. All were statistically significant.

Finnerty and Black-Scholes formulas, which were not crafted for DLOM estimation, understated DLOM by about 85% and 50%, respectively.

Exhibit 5[189] shows that the averages of the calculated DLOMs using the Finnerty formula range from 2.80–3.01 percent, and using the Black-Sholes formula they range from 9.11–9.36 percent. In contrast, the average reported discount of the 59-transaction population is 16.85 percent. It is unsubstantiated to say that the Finnerty and Black-Scholes formulas reliably predict DLOM.





[189] **(LINK TO COME)**.

The double-probability VFC Longstaff methodology predicted DLOM consistent with the discounts of the corresponding restricted stock transactions, resulting in an X-Y intercept of minus 2.84 percent, a coefficient of X of 1.0109, and an R-square of correlation of 54.19 percent. The test successfully falsified the formulas null hypothesis. The double-probability Finnerty and Black-Scholes tests failed to falsify the hypothesis, which adds credence to the reliability of the VFC Longstaff method for DLOM estimation.[190] It is error to say that the double-probability VFC Longstaff method did not reliably predict the set of corresponding restricted stock discounts and DLOM.

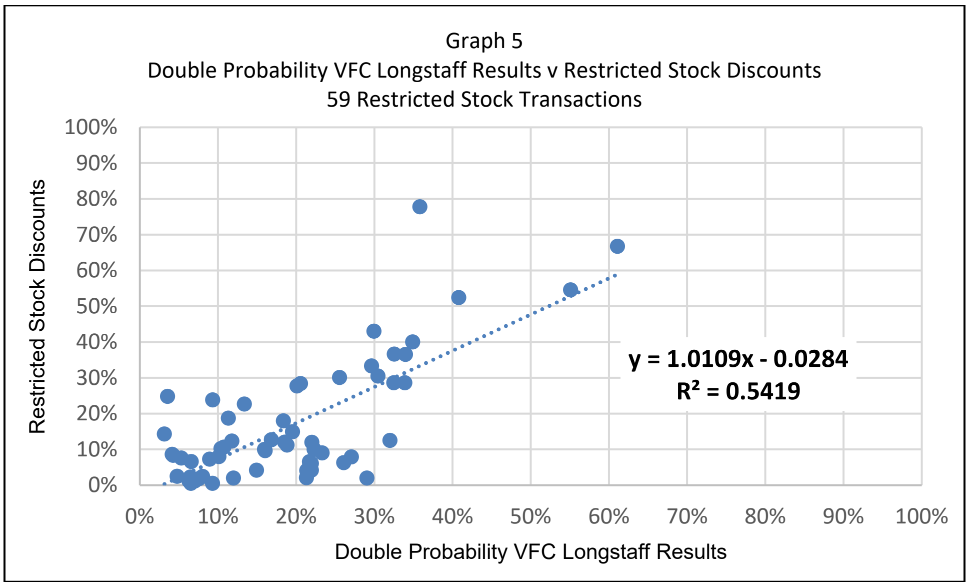


Exhibit 5 shows that the averages of the double probability DLOMs calculated using the VFC Longstaff methodology range from 19.48–19.96 percent. In contrast, the average reported discount of the 59-transaction population is 16.85 percent.[191]

[190] The formulas null hypothesis was falsified by the double-probability VFC Longstaff method regardless of the number of SIC code digits used to estimate the means and standard deviations of the SEC processing times for the 59-transaction population of restricted stocks. The double-probability Finnerty and Black-Scholes formula tests failed to falsify the hypothesis regardless of the number of SIC code digits used to estimate the means and standard deviations of the time variable. Exhibit 2 **(LINK TO COME)** shows the shifts in means and standard deviations of the SEC processing periods as the population of issuers is refined from 1 to 4 SIC code digits. Exhibit 5 **(LINK TO COME)** shows the resulting shifts in calculated DLOMs.

[191] The Empirical Research Table 8.4 discussion explains that further limiting the 59-transaction restricted stock population by removing those with discounts of 5% or less, a reduction to 45 transactions, results in an average restricted stock discount of 21.40% and an average double-probability VFC Longstaff DLOM of 21.61%.

Conclusion and Suggestions for Further Research

One set of assumptions used with the Black-Scholes formula and five sets of assumptions used with the VFC Longstaff method falsified the hypothesis, “The probability-based formula *does not predict* the corresponding observed restricted stock discounts with a linear regression relationship exhibiting these characteristics: (1) X-Y intercept ~ zero; (2) coefficient of X ~ Y; and (3) R-square > 50 percent.” All other DLOM methodologies discussed in this article failed to falsify the null hypothesis, and should be considered unreliable until scientifically shown otherwise.

It is error to say that the Black-Scholes test depicted by Graph 1 did not reliably predict the corresponding restricted stock discounts. However, because the Black-Scholes formula was not crafted to predict DLOM, no conclusion can be reached about its ability to predict DLOM.

It is error to say that the VFC Longstaff tests depicted by Graph 2 and Graph 5 and shown in Exhibit 5 do not reliably predict the corresponding restricted stock discounts. Additionally, the Longstaff formula was specifically crafted to estimate DLOM. Therefore, falsifying the formulas null hypothesis provides substantiation that the probability-based VFC Longstaff method predicts DLOM.

The single-probability VFC Longstaff test (Graph 2) is somewhat rigid, with DLOM calculations based on a static time period. That may be appropriate in some applications, such as a fixed-term lockup of a security. My preference for the typical fair market valuation of a nonmarketable security is to use the double-probability VFC Longstaff approach (Graph 5 and Exhibit 5) because it is dynamic. It more accurately reflects the world in which investments are made.

As shown above, there are significant issues regarding DLOM measurement and reliable DLOM estimation. The valuation profession would benefit from more extensive access to historical stock prices than were available to me. Access to significantly more price history would allow me to consider a greater population of transactions in the Pluris and FMV/Stout databases. The analysis discussed in this article would also be enhanced if a researcher undertook the task of repricing the Pluris transactions with attached warrants using the Black-Scholes option pricing formula. Finally, the profession would benefit from efforts by other researchers to justify DLOM methodologies using the scientific method applied to real world data, as this article has demonstrated. A simulation-derived analysis of DLOM estimation using the LEAPS methodology is suggested.

Chapter 12
CONCLUSIONS

Properly defined applications of the Longstaff, Black-Scholes, and Finnerty option formulas can result in DLOMs that correlate highly with combinations of price volatility and periods of illiquidity.[192] Double probability VFC DLOMs representing the combined probabilities of price and marketing period risks explained most of the variation the final set of analyzed restricted stock discounts. Double probability DLOMs calculated using the Longstaff formula provided values most consistent with the empirical evidence provided by the discounts of corresponding restricted stock transactions. The calculated DLOMs should be considered systematic.

The currently available empirical information supports the conclusion that double probability DLOMs calculated using the VFC Longstaff methodology results in reliable estimates of systematic DLOM. The final analyzed set of transactions was necessarily limited by the available data. The analyses presented in this research may be further refined if reliable price data before 2007 becomes available and as more recent restricted stock transactions become available to the author or other researchers.

Double probability DLOMs calculated using the VFC Black-Scholes formula tended to understate the discounts of restricted stock transactions by about 50%. Double probability DLOMs calculated using the VFC Finnerty formula tended to understate the discounts of corresponding restricted stock transactions by about 85%. However, both correlated highly with the reported discounts of the final set of analyzed transactions.

Important to valuation practitioners are the facts that (a) price data for currently listed public companies is available daily, and (b) time data is available for a great many publicly traded companies through the VFC public company filings database and for privately sold companies through the *BIZCOMPS*® and *DealStats*® databases. These factors allow practitioners to calculate DLOMs using the option models that are highly specific to the valuation subject and the valuation date. The research supports a conclusion that more objective, persuasive, and reliable DLOMs can be developed using the VFC DLOM Calculator® methodology than by using previously available methodologies.

Marc Vianello, CPA, ABV, CFF
July 1, 2019

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[192] Although not discussed in this research paper, the Finnerty formula was studied in the same manner as the Longstaff and Black-Scholes formulas. The Finnerty formula yielded comparable double probability regression results except for understating the reported restricted stock discounts of the final study set by a factor of about 7.